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A review on design criteria for vortex tubes

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Abstract In this study, the past investigations of the design criteria of vortex tubes were overviewed and the detailed information was presented on the design of them. Vortex tubes were classified and the type of them was described. All criteria on the design of vortex tubes were given in detail using experimental and theoretical results from the past until now. Finally, the criteria on the design of them are summarized.

List of symbols

A	cross section (m^2)
CFD	computational fluid dynamics
COP	coefficient of performance
c_p	specific heat at constant pressure (kJ kg^{-1} K^{-1})
c_v	specific heat at constant volume (kJ kg^{-1} K^{-1})
d	diameter (m)
D	diameter (m)
h	enthalpy (kJ kg^{-1})
k	specific heat ratio
k	Boltzmann constant (J K^{-1})
L	length (m)
$\dot{m}$	mass flow rate (kg s^{-1})
N	number
p	pressure (Pa)
$\dot{Q}$	heat transfer rate (W)
R	specific gas constant (kJ kg^{-1} K^{-1})
RHVT	Ranque–Hilsch vortex tube
S	entropy (W K^{-1})
T	temperature (K)
T^*_{sm}	temperature assumed to be $T_h^{1-\varepsilon}T_c^{\varepsilon}$
$\dot{W}$	power (W)
X	normalised pressure drop ($X = (p_{in} - p_c)/p_{in}$)

Greek symbols

α	angle of cone-shaped control valve
α	ratio of hot end area to tube area
β	cold orifice diameter ratio ($\beta = d_c/D$)
ε	cold fraction
ε_o	Lennard–Jones potential
η	efficiency
ΔT	temperature difference
$\Delta T/T_{in}$	normalised temperature drop/rise
Γ	$(k-1)/k$
Θ	irreversibility parameter
τ_p	pressure ratio ($= p_{in}/p_c$)

Subscripts

atm	atmosphere
c	cold
cr	cooler
cr	critical
h	hot
hp	heat pump
in	inlet
i	irreversible

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1 Introduction

The vortex tube is a device without moving mechanical parts, which converts a gas flow initially homogeneous in temperature, into two separate flows of differing temperatures. It separates compressed gas stream into a low total temperature region and a high one. Such a separation of the flow into regions of low and high total temperature is referred to as the temperature (or energy) separation effect. The vortex tube contains the following parts: one or more inlet nozzles, a vortex chamber, a cold-end orifice, a hot-end control valve and a tube. In general, the vortex tube has been known by different names. The most well-known names are: vortex tube, Ranque vortex tube (first discoverer), Hilsch vortex tube or Ranque–Hilsch vortex tube (who improved the performance of the vortex tubes after Ranque), and Maxwell–Demon vortex tube (derived from the name of Maxwell and Demon group who together studied the molecule of hot air moving within the tube) [1, 2]. Although there are various names, only ''RHVT'' will be used in this article.

The vortex tube effects were first observed by Ranque, a French metallurgist and physicist about 1930. He formed a small company to exploit the item but it soon failed. He presented a paper on the vortex tube to a scientific society in France in 1933, but it was met with disbelief and disinterest [3, 4]. Thereafter, the vortex tube disappeared for several years, until Rudolph Hilsch studied it and published his findings in the mid-1940s [5]. In 1947, Hilsch systematically examined the effect of the inlet pressure and the geometrical parameters of the vortex tube on its performance and presented a possible explanation of the energy separation process. In 1954, Westley [6] published a comprehensive survey entitled "A bibliography and survey of the vortex tube", which included over 100 references. In 1951 Curley and McGree [7], 1956 Kalvinskas [8], in 1964 Dobratz [9], in 1972 Nash [10], in 1979 Hellyar [11] made important contribution to the RHVT literature by their extensive reviews on the vortex tube and applications. It is important to note that the investigations by Gao [1], Fulton [12, 13], Hartnett and Eckert [14], Martynovskii and Alekseev [15], Lay [16, 17], Deissler and Perlmutter [18, 19], Takahama and Takahama et al. [20–25], Sibulkin [26], Linderstrom-Lang [27–30], Borisenko et al. [31], Bruun [32], Raiskii and Tunkel [33], Soni [34], Soni and Thompson [35], Marshall [36], Kurosaka [37], Stephan et al. [38, 39], Balmer [40], Nabhani [41], Ahlborn et al. [42–46], Gutsol [47], Gutsol and Bakken [48], Lewins and Bejan [49], Saidi and Yazdi [50], Cockerill [51], Khodorkov et al. [52], Poshernev and Khodorkov [53, 54], Shannak [55], Singh et al. [56], Behera et al. [57], Aljuwayhel et al. [58], Skye et al. [59], Eiamsa-ard and Promvonge [60–63], Azarov [64–68] made important contribution on the vortex tube knowledge.

Since its discovery by Ranque the RHVT has been the subject of considerable interest both from the theoretical and practical application standpoints. The effect of energy separation (temperature stratification) and other effects arising in vortex tubes allow them to be widely used for different purposes. That vortex tubes have many advantages makes them very attractive for industrial applications. Vortex tubes have widely been used in various applications where compactness, safety, and low equipment cost are basic factors: heating and cooling applications, gas liquefication, separation of gas mixtures, drying of gases, using in chemical industry, electric production, snow making etc. Nowadays, RHVTs are produced by different commercial companies with a wide range of applications [69, 70].

Despite the interest created by these applications, the underlying mechanics lying on the temperature separation are not clearly understood. The mechanism underlying the energy transfer from the cold to the hot flow remains elusive, however. There is debate even as to the basic physics of the phenomenon. According to Cockerill [51] one obstacle to investigations of the Ranque–Hilsch tube is the lack of any substantial published literature review since Westley in 1954. He emphasises that an important task early in any study of the vortex tube must be to draw together the mass of literature. Eiamsa-ard and Promvonge [2] presents an overview of the phenomena occurring inside the vortex tube during the temperature/energy separation on both the counter flow and parallel flow types. The review includes important definitions, classification and parametric study of the vortex tube, experimental and qualitative, analytical and numerical work. A short observation is also presented in the paper. However, it is difficult to design a vortex tube with definite integral characteristics for a concrete application because the available experimental data are not clearly understood and there are no entirely correct generalizations. This is because available experimental data cannot be completely understood and it is not possible to generalize all of these data [71]. This reveals the important of a review on vortex tube and design criteria including the last and present-day investigations.

The purpose of this article is to overview of the past and present-day investigations of the design criteria of vortex tubes and to provide collectively detailed information on the design of vortex tubes. The article is separated into five sections. Section 2 presents the explanations of some important terms commonly used in the RHVT work. Section 3 describes the classification of the RHVT. Section 4 presents the survey of the past and present-day research on

the RHVT, emphasizing the design criteria. The design criteria deducted from the experimental and theoretical RHVT investigations are tabulated and summarized in the final section.

2 Definitions

This section presents a few important terms commonly used in vortex tube work.

2.1 Cold flow mass ratio (cold mass fraction)

The cold flow mass ratio (cold mass fraction) is the most important parameter indicating the vortex tube performance and the temperature/energy separation inside the RHVT. The performance of the RHVTs is evaluated based on the cold fraction. The cold mass fraction is the percentage of input compressed air that is released through the cold end of the tube. It is the mass flow rate of cold gas divided by mass flow rate of the inlet gas:

$$\varepsilon = \frac{\dot{m}_c}{\dot{m}_{in}} \tag{1}$$

where $\dot{m}_c$ represents the mass flow rate of the cold stream released, $\dot{m}_{in}$ represents the inlet or total mass flow rate of the pressurized inlet working fluid. Therefore, ε changes in the range $0 \le \varepsilon \le 1$.

2.2 Cold and hot temperature difference

Cold temperature difference or temperature reduction is defined as the difference in temperature between inlet flow temperature and cold flow temperature:

$$\Delta T_c = T_{in} - T_c \tag{2}$$

where T_{in} is the inlet flow temperature and T_c is the cold flow temperature. Similarly hot temperature difference is defined as

$$\Delta T_h = T_h - T_{in} \tag{3}$$

2.3 Normalised temperature drop/rise

Normalised cold temperature drop is defined as the ratio of cold temperature difference to inlet temperature:

$$\frac{\Delta T_c}{T_{in}} = \frac{(T_c - T_{in})}{T_{in}} \tag{4}$$

Similarly normalised hot temperature rise is defined as

$$\frac{\Delta T_h}{T_{in}} = \frac{(T_h - T_{in})}{T_{in}} \tag{5}$$

2.4 Cold orifice diameter

Cold orifice diameter ratio (β) is defined as the ratio of cold orifice diameter (d_c) to vortex tube diameter (D):

$$\beta = \frac{d_c}{D} \tag{6}$$

2.5 Isentropic efficiency

Assuming the process inside RHVT as isentropic expansion, isentropic efficiency is [1, 51, 52, 72]

$$\eta_{is} = \frac{h_{in} - h_c}{h_{in} - h_s} \tag{7}$$

where h_{in} is enthalpy at the inlet to vortex tube, h_c cold exhaust enthalpy, and h_s enthalpy after isentropic processes. For an ideal gas

$$\eta_{is} = \frac{T_{in} - T_c}{T_{in} - T_s} \tag{8}$$

For an isentropic expansion, the exhaust temperature is

$$T_s = T_{in}\left(\frac{p_c}{p_{in}}\right)^{\frac{(k-1)}{k}} \tag{9}$$

Introducing Eq. (9) into Eq. (8) yields

$$\eta_{is} = \frac{T_{in} - T_c}{T_{in}\left[1 - (p_{atm}/p_{in})^{(k-1)/k} - 1\right]} \tag{10}$$

where η_{is}, p_{in}, p_{atm} and k are the isentropic efficiency, inlet air pressure, atmosphere pressure and specific heat ratio, respectively.

2.6 Coefficient of performance

The coefficient of performance (COP) as a refrigerator is defined as the ratio of the cooling power gained by the system to the work power [73]

$$\mathrm{COP}_{cr} = \frac{\dot{Q}_c}{\dot{W}} \tag{11}$$

Here the cooling power can be calculated according to the cooling capacity of the cold exhaust gas (e.g. the heat necessary to heat up the cold exhaust gas from the cold exhaust temperature to the applied temperature) [1]:

$$\dot{Q}_c = \dot{m}_c c_p (T_{in} - T_c) \tag{12}$$

In a conventional refrigeration system, there is a compressor, so the work power is the input power of the compressor. But in the RHVT system, usually a compressed gas source is used, so it is not easy to define the work power. By analogy the work used to compress the

gas from the exhaust pressure up to the input pressure with a reversible isothermal compression process [1].

$$\mathrm{COP}_{\mathrm{cr}} = \frac{k}{k-1}\frac{\varepsilon(T_{\mathrm{in}} - T_{\mathrm{c}})}{T_{\mathrm{in}} \ln \frac{p_{\mathrm{in}}}{p_{\mathrm{c}}}} \tag{13}$$

The coefficient of performance (COP) for a heat pump is the ratio of the energy transferred for heating to the work power [73]

$$\mathrm{COP}_{\mathrm{hp}} = \frac{\dot{Q}_{\mathrm{h}}}{\dot{W}} \tag{14}$$

For the RHVT system, the heating power can be expressed as the heating capacity of the hot exhaust gas

$$\dot{Q}_{\mathrm{h}} = \dot{m}_{\mathrm{h}} c_{\mathrm{p}} (T_{\mathrm{h}} - T_{\mathrm{in}}) \tag{15}$$

The work power used by the system is taken the same as used above for the refrigerator. So the coefficient of performance of the RHVT as a heat pump is [1]

$$\mathrm{COP}_{\mathrm{hp}} = \frac{k}{k-1}\frac{(1-\varepsilon)(T_{\mathrm{h}} - T_{\mathrm{in}})}{T_{\mathrm{in}} \ln \frac{p_{\mathrm{in}}}{p_{\mathrm{c}}}} \tag{16}$$

2.7 Irreversibility parameter

The irreversibility parameter is the dimensionless entropy generation from the irreversible processes, and defined as [1]

$$\Theta_{\mathrm{ir}} = \frac{\dot{S}_{\mathrm{i}}}{\dot{m}_{\mathrm{in}} R_{\mathrm{m}}} = \frac{1}{\Gamma} \ln \frac{T^{*}_{\mathrm{sm}}}{T_{\mathrm{in}}} + \ln \frac{p_{\mathrm{in}}}{p_{\mathrm{atm}}} = \ > 0 \tag{17}$$

where $\dot{S}_{\mathrm{i}}$ is the entropy production rates due to irreversible processes, R_{m} is the specific gas constant, $\Gamma = (k-1)/k$, and T^{*}_{sm} is the temperature assumed to be $T_{\mathrm{h}}^{1-\varepsilon} T_{\mathrm{c}}^{\varepsilon}$.

3 Types of vortex tubes

Vortex tubes are classified by their main technological and design features: flow configuration, the method of heat supply (removal), and how removal of low-pressure gas streams is organized. Table 1 presents the types of vortex tubes. For the positioning of the cold exhaust, there are two different types: counterflow vortex tubes and parallel flow (uniflow) vortex tubes. Vortex tubes are classified as uncooled (adiabatic) and cooled (nonadiabatic) according to the method of heat supply (removal). On the other hand according to the how removal of low-pressure gas streams is organized vortex tubes are called as dividing vortex tubes, self-evacuating vortex tubes, and vortex ejectors.

3.1 Counterflow vortex tubes

In counterflow vortex tubes the cold exhaust is placed on the other side from the hot exhaust, as shown in Fig. 1.

Table 1 Classification of vortex tubes [52]

Method	Classification
Flow characteristics	Parallel flow (uniflow) vortex tubes
	Counterflow vortex tubes
The method of heat suppy (removal)	Uncooled (adiabatic) vortex tubes
	Cooled (nonadiabatic) vortex tubes
How removal of low-pressure gas streams is organized	Dividing vortex tubes
	Self-evacuating vortex tubes
	Vortex ejectors

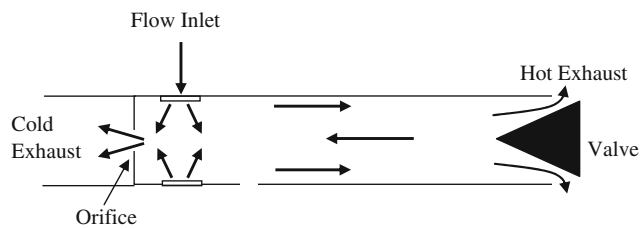

Fig. 1 Counterflow vortex tube

The working gas is tangentially injected into vortex tube via inlet nozzles positioned next to the cold exhaust. A strongly swirling flow is created and the gas proceeds along the tube. The outer region of the flow is found to be warmer than the inlet gas, while gas towards the centre of the tube experiences cooling. Part of the gas in the vortex tube reverses for axial component of the velocity, and it moves from the hot end to the cold end. An orifice positioned just behind the flow inlets separates the cool central gas, which then exits the tube at the left-hand side. The warm peripheral flow leaves at the right-hand side of the tube, where a valve is positioned to allow regulation of the relative quantities of hot and cold gas [1, 51].

3.2 Uniflow vortex tubes

When the cold exhaust is placed at the same side of the hot exhaust, it is named "uniflow (or parallel flow) vortex tubes". The fundamental aspects of this configuration are the same as for the counterflow tube. Its distinguishing features are that the orifice and valve are combined at one end of the tube, while other end of the tube, adjacent to the inlet nozzles is sealed (Fig. 2). Many investigators have suggested that uniflow tubes perform less well than equivalently proportional counterflow designs. So, most of the time, the counterflow geometry has been chosen [1, 51].

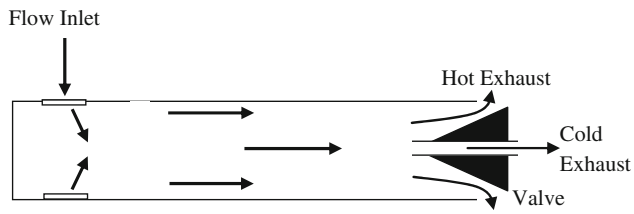

Fig. 2 Parallel flow vortex tube

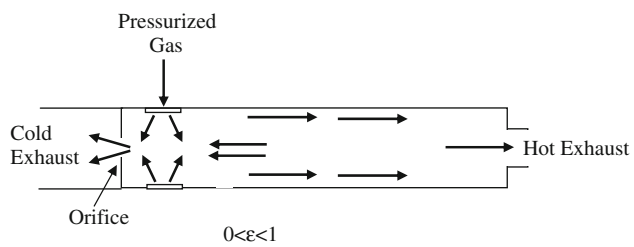

Fig. 3 Dividing vortex tube

3.3 Dividing vortex tubes

Figure 3 shows a dividing vortex tube schematically. The dividing vortex tube is the best-known and most widespread type of RHVTs. It has both cold and hot flow. It has up to ten designs, as shown in Fig. 4, and is used on various industries [64–68].

3.4 Uncooled (adiabatic) vortex tubes

Adiabatic vortex tubes are ones heat transfer to environment are neglected.

3.5 Cooled (nonadiabatic) vortex tubes

Nonadibatic vortex tubes are ones heat transfer from the hot fluid to a cooling fluid occurs. These tubes are also called "cooled vortex tube". Figure 5 shows such a cooled vortex tube schematically. The cooled vortex tube differs from the dividing vortex tubes in that its hot end is closed, it is fitted with an outer jacket which is fed a cooling fluid, and all the gas entering the nozzle inlet emerges cooled (20–30 K) through the diaphragm aperture, i.e. in the given case $\varepsilon = 1$. The cooling vortex tube does not produce strong cooling effects. It is distinguished from other vortex tubes; however, by maximum cooling power, which allows it to be used most efficiently at the higher temperature level in a combined regenerative throttling cycle [52].

3.6 Dividing vortex tube with an additional stream

Another configuration of the dividing vortex tube is one with an additional stream, as presented in Fig. 6. At the hot end, in the center of the control valve, there is an orifice which allows feedback gas to be injected into the vortex tube. These tubes are also called as double circuit vortex tubes. First circuit consists of a peripheral vortex circuit with the working fluid entering the tube through the nozzles. Second circuit is an axial vortex circuit composed of additional gas entering vortex tube through the orifice at

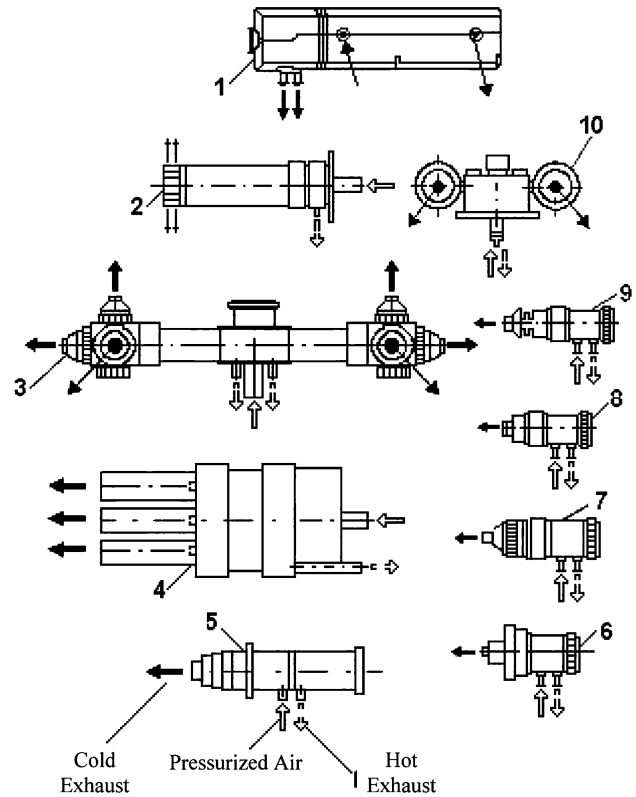

Fig. 4 Various dividing vortex tubes [66]

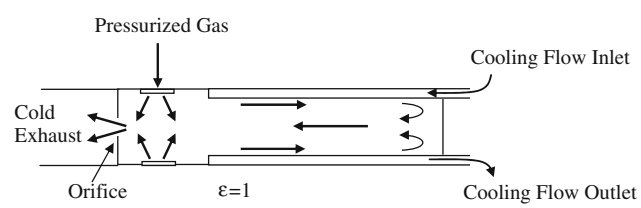

Fig. 5 Cooled vortex tube

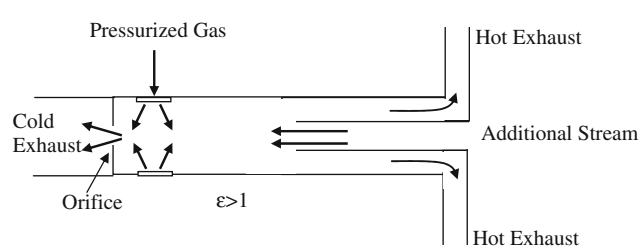

Fig. 6 Dividing vortex tube with an additional stream

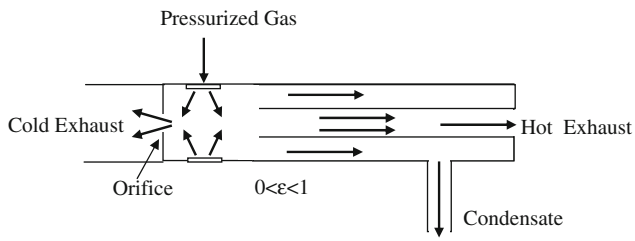

Fig. 7 Triple-stream vortex tube

the hot end. The dividing vortex tube with an additional stream ensures that $\varepsilon > 1$, provided that additional gas at a low excess pressure is fed from the hot end in the reduced pressure zone on the axis. The dividing vortex tube with an additional stream clearly demonstrated that energy exchange can occur between the peripheral vortex and the non-twisted axial gas flow with a fairly strong cooling effect: the adiabatic efficiency is 0.36 at an outlet pressure of 0.4 MPa; this is 30% higher than the results obtained on vortex tubes of a different design. This design allows to increase the cooling power of the system and to enhance the performance of the vortex tube [52, 74].

3.7 Triple-stream vortex tube

The triple-stream vortex tube, in contrast to an ordinary dividing vortex tube, has an internal cylinder (tube) that forms with the casing an annular gap into which is fed the condensate that is thrown to the periphery (Fig. 7). The amount of condensate separated directly in the triple-stream vortex tube may be 40–90% of the total condensate in the gas. This indicates the high efficiency of the tube as a separator [52].

3.8 Self-evacuating vortex tube

The self-evacuating vortex tube is characterized by the maximum cooling effect (of all vortex tubes) because of a pressure drop in the paraxial zone by installing one or two non-twisted slot diffusers at the hot end (Fig. 8). The self-evacuating vortex tube does not allow a cold gas stream to be emitted into the environment and is intended primarily for deep cooling of cylindrical bodies [52].

3.9 Vortex ejectors

The vortex ejector, given certain geometric parameters and gasdynamic conditions, can produce and maintain a pressure below the atmosphere in the paraxial zone. Vortex ejectors (Fig. 9) differ from conventional direct-jet ejectors in that they have a smoother characteristic and maintain sufficient efficiency over a wide range of variation of the initial parameters [52].

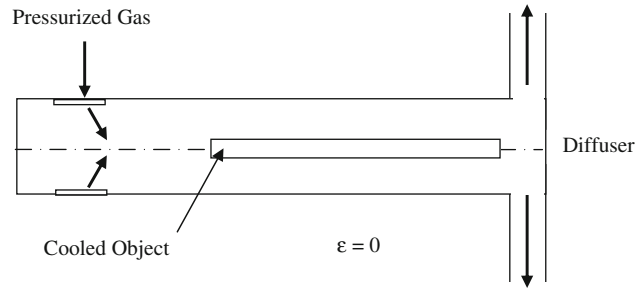

Fig. 8 Self-evacuating vortex tube

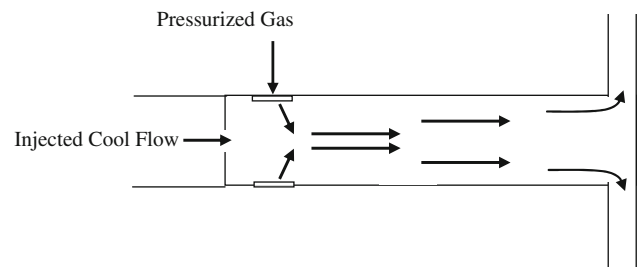

Fig. 9 Vortex ejector

4 Design criteria for vortex tubes

Many variables influence the flow and performance of RHVTs: geometrical parameters, mass flows, reservoir conditions, gas properties, internal flow parameters, and other factors [51, 75]:

- Geometrical parameters: hot and cold tube length, internal diameter of tube, effective diameter of inlet nozzles, number of inlet nozzles, orientation of inlet nozzles, cold orifice diameter, shape and length of hot end valve, effective diameter of hot flow exit restriction, vortex chamber etc.
- Mass flows: cold mass fraction, overall mass flow rate.
- Reservoir conditions: inlet (reservoir) pressure, inlet (reservoir) temperature, gas density at tube inlet etc.
- Gas properties: gas viscosity, gas thermal conductivity, heat capacity of gas at constant pressure, gas isentropic exponent, gas coefficient of thermal expansivity, mole fraction of gas component i in any mixture, at the inlet etc.
- Internal flow parameters: static pressure at cold exit, static pressure at hot exit, swirl velocity at tube inlet etc.
- Other factors: material of tube, internal roughness, gas molecular mass etc.

As seen, in order to design a good RHVT, the inlet nozzle, the vortex chamber, the cold orifice, the hot and cold tube length, the tube geometry, the tube material, the fluid properties, the cold mass fraction etc., are all relevant

design parameters. In this section, the design criteria for RHVT are reviewed and discussed.

4.1 Hot and cold tube length

There have been many investigations studying the effect of length of vortex tube on the performance characteristics. All authors found that an efficient tube of either design should be many times longer than its diameter. The earlier vortex tubes generally had hot tubes with lengths of about $50D_h$. For example Hilsch [5] suggested that L/D_h should be around 50 for good temperature separation. According to Westley [76] the only requirement is that the tube exceeds $10D_h$. Martynovskii and Alekseev [15] emphasized that the length and geometry of the RHVT is significantly important to obtain hot flow temperature and there is an optimum L/D ratio. They determined that increasing the length of an optimum designed RHVT increases efficiency and optimum performance is obtained in RHVT with length of $40 < L/D < 50$. Lay [16, 17] reported that Ranque–Hilsch effect is not important for L/D greater than 9. Gulyaev [77, 78] determined that the minimum length for cylindrical hot tube was about $10D_h$. If the hot tube is conical, rather than cylindrical, the minimum length must be incresaed, to about $13D_h$ for a tube with an angle of divergence of 2°–3°. Linderstrom-Lang [28] observed that when using short vortex tubes temperature separation is small and fluid separation is high, when using long vortex tubes temperature separation is high and fluid separation is small. Lewellen [79] stated that "as long as the tube wall is insulated the temperature separation in the tube should be unaffected by L_h/D_h as long as some minimum length is exceeded." Raiskii and Tunkel [33] and Soni and Thompson [35] found that L/D_h should be greater than 45. Takahama and Yokosawa [24] suggested using a tube length $L \geq 100D_{vt}$ in order to obtain a better performance. Amitani et al. [80] indicated that the shortened vortex tube of six tube diameters length had the same efficiency as a longer and smaller vortex tube when perforated plates are equipped to stop the rotation of the stream in the tube. Saidi and Yazdi [50] found that increasing tube length increases temperature differences and decreases exergy destruction. For $L/D \leq 20$ energy separation was quite low. For $L/D \geq 55$ the variation of efficiency with L/D was not considerable. According to these results Saidi and Valipour [81] concluded that the optimum value of L/D is in the range of 20–55. Singh et al. [56] reports that "length of the tube has no effect on the performance of the vortex tube in the range of 45–55 (L/D_T ratio)". Behera et al. [57] presented that increase in the length of tube enhances the temperature separation up to the condition that stagnation point is within the length of tube. The investigations had shown that L/D ratio in the range of 25–35 is optimum for achieving best thermal performance for 12 mm vortex tube. Flow within the RHVT has forced and free vortex until to the point of stagnation in the tube. A CFD investigation by Aljuwayhel et al. [58] shows that increasing the length of the vortex tube from 10 to 30 cm results in an increase in the temperature drop of the cold flow of 0.7 K, or 2.6%. However, a further increase in length to 40 cm does not change the energy separation; the cold outlet temperature for the 40 cm length vortex tube is the same, to within 0.01 K, as that of the 30 cm length vortex tube. Aljuwayhel et al. [58] reported that there is a critical length of the vortex tube over which the majority of the energy transfer takes place; under the conditions used in the CFD model this length is nominally 22.5 cm. This is both the length that the cold flow travels along the vortex tube before returning to the cold end, and the length over which the majority of the energy transfer has occurred. According to Gao [1], the result of the study of the influence of the tube length suggests that a tube length of 2,586 mm is optimal, corresponding to L/D_{vt} is about 65 (no larger L/D_{vt} ratios were tested). In an experimental study, performance of counter flow type RHVT, with a length to diameter ratio of 10, 15 and 18, were investigated with 2, 4, 6 nozzles [82]. It was concluded that the best performance was obtained when the ratio of vortex tube's length to the diameter was 15 and the nozzle number was at least four, and the inlet pressure was as high as possible. The magnitude of the energy separation increases as the length of the vortex tube increases to a critical length; however, a further increase of the vortex tube length beyond the critical length does not improve the energy separation. This emphasizes that one of the most important factors for the energy separation process in RHVTs is the stagnation point. Very long vortex tubes affect the stability of the stagnation point negatively. Aydın and Baki [83] performed the RHVT experiments in tubes with inner diameter of 18 mm made of aluminum/stainless steel. Six different RHVTs having various lengths were used: 250, 350, 450, 550, 650 and 750 mm. The tube with $L = 350$ mm presented the optimum results for the highest possible temperatures ($L/D \sim 20$).

As a result from the literature survey it may be emphasized that the length of vortex tube affects performance significantly, optimum L/D is a function of geometrical and operating parameters, and L/D has no effect on performance beyond $L/D > 45$.

4.2 Vortex tube diameter

Different vortex tube diameters have been used in experimental RHVT investigations, from diameters as low as 4.4 mm and as high as 800 mm [2]. Commercial vortex tube companies manufacture vortex tubes with diameter

from 2.7 to 80 mm [84–93]. On the other hand, the vortex tubes used for gas liquefication and separation can have much greater diameters. Instead of studying the effect of the diameter of vortex tube directly, the effect of *L/D* on performance has usually been preferred in experimental investigations. A numerical model predicts that the reduction in the diameter to 1.5 cm increases the cold temperature drop by 1.2 K, or 4.4%, compared to the 2 cm diameter model. An increase in the diameter to 3 cm decreases the cold temperature drop by 6.5 K, or 24%. The reduction of the energy separation with increasing diameter is directly related to the magnitude of the gradients of the angular velocity. Because the same inlet boundary conditions (the inlet area, stagnation pressure, stagnation temperature, and velocity components) are used, the magnitude of the gradients in the angular velocity is much lower in the larger diameter case. Because the angular velocity gradients give rise to tangential work transfer, the energy separation is reduced. This suggests that in general smaller diameter vortex tubes will provide more temperature separation than larger diameter ones [58].

For fixed inlet conditions (supply pressure) a very small diameter vortex tube would offer considerably higher back pressures and, therefore, the tangential velocities between the periphery and the core would not differ substantially due to the lower specific volume of air (still high density) while the axial velocities in the core region are high. This would lead to low diffusion of kinetic energy which also means low temperature separation. On the other hand, a very large tube diameter would result in lower overall tangential velocities both in the core and in the periphery region that would produce low diffusion of mean kinetic energy and also low temperature separation [2].

4.3 Diameter/area of inlet nozzles

The inlet nozzle is an important component of the RHVT. In order to get the best performance of the RHVT, the pressure loss over the inlet nozzle should be as small as possible, the Mach number at the exhaust of the inlet nozzle should be as high as possible, and the momentum flow at the exhaust should be as large as possible. It is clear that the geometry of the inlet nozzle can influence the exit gas properties [1]. A very small inlet nozzle would give rise to considerable pressure drop in the nozzle itself, leading to low tangential velocities and hence low temperature separation. A very large inlet nozzle would fail to establish proper vortex flow resulting again in low diffusion of kinetic energy and therefore low temperature separation. The inlet nozzle location should be as close as possible to the orifice to yield high tangential velocities near the orifice. A nozzle location away from the orifice would lead to low tangential velocities near the orifice and hence low temperature separation [2].

There have been many investigations aiming to study the effect of the geometry of the inlet nozzle on the performance of the RHVT. In these investigations some investigator presented the results using the nozzle diameter, some the ratio of the inlet nozzle area to the cross section of the vortex tube (A_{in}/A_{vt}). Westley [94] experimentally optimized the geometry of a RHVT system and found that the ratio of the flow area of the inlet nozzle to the flow area of the vortex tube should be as follows:

$$\frac{A_{in}}{A_{vt}} \cong 0.156 + 0.176/\tau_p, \quad \tau_p = \frac{p_{in}}{p_c} = 7.5 \tag{18}$$

where A_{vt} is the flow area of the vortex tube, A_{in} is the flow area of the inlet nozzle, p_{in} is the inlet pressure and p_c is the cold exhaust pressure. Martynovskii and Alekseev [15] found that optimum nozzle diameter is a function of the diameter of the vortex tube, and the nozzle diameter should be increased as the diameter of the vortex tube increases for optimum results. The effect of different inlet nozzle diameters, at the optimum cold gas flow ratio and cold gas orifice diameter, on the maximum temperature drop was studied by Westley [76]. The temperature drop ratio,$(T_{in} - T_c)/T_{in}$, for a $d_{in}/D_{vt} = 0.266$ was found to increase with the pressure ratio until it reaches an asymptotic value greater than that for the other diameter ratios. The temperature drop ratio for values of p_{in}/p_c greater than 11 or 12 increased more slightly. According to Takahama [20] the geometry should have the following relationship in order to have larger temperature differences or larger refrigeration capacity:

$$\begin{aligned} d_{in}/D_{vt} \le 0.2; \quad Nd_n^2/D^2 = 0.16 - 0.20 \\ d_c < D - 2d_n; \quad d_c^2/Nd_n^2 \le 2.3 \end{aligned} \tag{19}$$

where d_{in} is the diameter of the inlet nozzle, D_{vt} is the diameter of the vortex tube, d_c is the diameter of the cold orifice, and N is the nozzle number. In another investigation it was concluded that increasing the effective height (vertical size) of the inlet nozzle results in an increase in the temperature differences between the hot and cold flow temperature [26]. Linderstrom-Lang [30] found experimentally that optimum inlet nozzle should be approximately $0.25D_{vt}$. Soni [34] proposed that the nozzle be designed in according to the following relationship:

$$\frac{A_{in}}{A_{vt}} = 0.084 \approx 0.11 \tag{20}$$

In an analysis using exergy model of a vortex tube system, second law efficiency improved and exergy destruction decreased with the increasing nozzle diameter [50]. Aydın and Baki [83] observed experimentally that the optimum nozzle diameter was $d/D = 1/3$ in a counterflow RHVT.

All these investigations prove that increasing nozzle diameter, generally, increases the performance and the optimum nozzle diameter is about to be 0.25 D_{vt}.

4.4 Type and number of inlet nozzles

In vortex tubes, types of vortex chambers, types and number of inlet nozzles are quite important. Many investigations have been carried out on different types and number of nozzles to arrive at the optimum profile of the inlet nozzle and number of nozzle(s). All investigations reported that the inlet nozzles should be designed so that the flow be tangentially into vortex tube. Most vortex chambers are circular with a single circular nozzle inlet. Martynovskii and Alekseev [15] designed three different chamber configurations, including one called a "Hilsch Whorl", and concluded that a circular chamber with two nozzles was the most efficient. In 1957, Westley [76] used multiple rectangular nozzles entering a circular chamber. Reynolds [95–97] suggested that the inlet nozzle should be in the form of a slot. Reynolds suggested having more slots in the RHVT. He noted that more slots do improve the performance as well. Metenin [98] used six tangential nozzles and in 1964 he designed one nozzle called as Archimedian spiral, as shown in Fig. 10c, [99]. Parulekar [100] suggested that the designs of the vortex chamber and the inlet nozzle are very important, and he mentioned that the inlet nozzle should have an Archimedian spiral shape and its cross section should be slotted. A single rectangular nozzle was used by Leites et al. [101] on their large industrial vortex tube. Saidi and Valipour [81] conducted experimental investigations using vortex tubes with three and four nozzles. The results showed the nozzle with three intakes presents better performance than four intakes nozzle from the point of view of refrigeration efficiency. Promvonge and Eiamsa-ard [60] experimentally studied the energy and temperature separations in the vortex tube with a snail entrance. In their experimental results, the use of snail entrance could help to increase the cold air temperature drop and to improve the vortex tube efficiency in comparison with those of original tangential inlet nozzles. In another investigation Promvonge and Eiamsa-ard [61] concluded that "the increase of the number of inlet nozzles led to higher temperature separation in the vortex tube". Behera et al. [57] suggest that optimum nozzle profile and number can be determined by using CFD analysis. They used five different nozzle configurations in a vortex tube of 12 mm diameter: two numbers of convergent nozzles, single helical circular nozzle, single helical rectangular nozzle, straight six numbers of nozzles, and convergent six numbers of nozzles. The performance of the nozzles has been characterized by the magnitude of swirl velocity and radial symmetry of flow. Among them the nozzle configuration with six numbers of convergent nozzles yielded the highest swirl velocity, and radial symmetry of flow was obtained along with good swirl velocity, which resulted in the highest total temperature difference compared to all the nozzle profiles. Two typical nozzle geometries were studied by Gao [1]: a conical nozzle and a convergent (linear) slot nozzle with constant width. He determined that the optimization occurs by using a short slot and more number of slots. According to Gao [1] applying a slot-ring instead of the nozzle ring, using a large number of slots and using a larger exhaust tube to decrease the exhaust pressures all can increase the temperature differences and improve the performance. Dincer et al. [82] conducted experimental investigation in vortex tubes with 15 L/D ratio. The vortex tubes with 4 and 6 nozzles yields better performance than the vortex tube with 2 nozzles. The vortex tube with 4 nozzles gave the best performance. In 2007, Wu et al. [102] used a new nozzle with equal Mach number gradient and an intake flow passage with equal flow velocity in the modified vortex tube (Fig. 10b). The experimental results indicated that the cooling effect of the improved nozzle is about 2.2°C lower than that of the nozzle with normal rectangle and even 5°C lower than that of the nozzle with Archimedes' spiral.

Some investigations that study the effect of nozzle number on the performance of RHVT suggest that increasing nozzle number improves temperature separation; others suggest that the performance of RHVT decreases with increasing nozzle number. According to the first group investigators increasing nozzle number results in flow to accelerate and flow rate to increase. Strong swirling flow inside the vortex tube is created. In addition this causes high friction dissipation between the flow layers and high momentum transfer from central to circumferential regions. Finally flow temperature in central region decreases and those in circumferential regions increases. According to other investigators as nozzle number increases the flow inside the vortex tube becomes more turbulent due to interactions of the inlet flows, hot and cold flow mixes and thus temperature separation decreases.

4.5 Cold orifice

The optimum cold orifice diameter is a function of the gas flow rate through the orifice; hence it is a function of the pressure drop across the orifice and the cold gas mass flow ratio. If the orifice is too large, hot gas from the outer edge of the vortex is drawn through the orifice with the colder inner gas. Therefore, a very large cold orifice would tend to draw air directly from the inlet and yield weaker tangential velocities near the inlet region, resulting in low temperature separation. However, if the orifice diameter is too small, there is a significant pressure drop across the orifice,

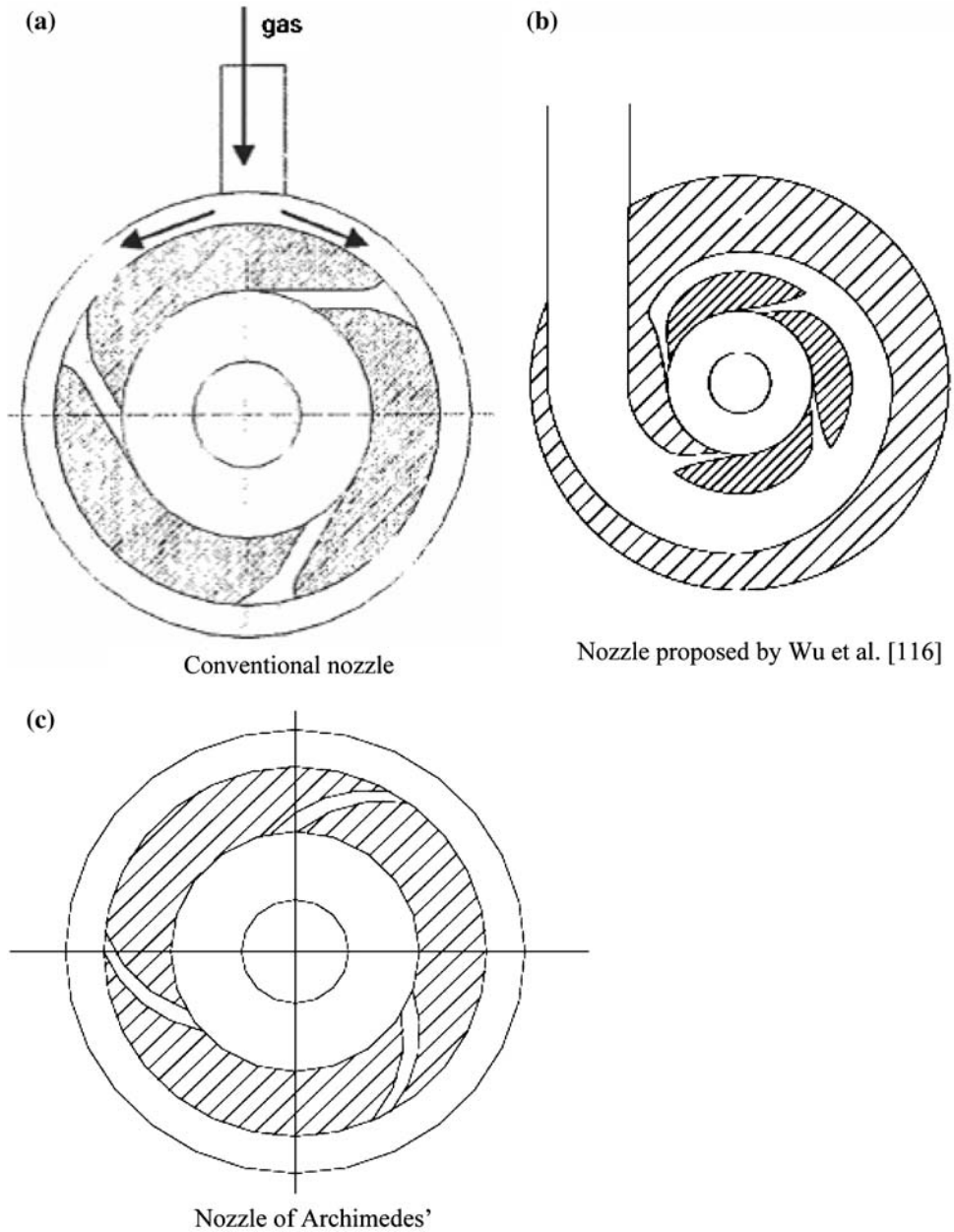

Fig. 10 Some nozzle configurations [102]

and hence a smaller pressure drops across the inlet gas nozzle. Therefore, a very small cold orifice would give higher back pressure in the vortex tube, resulting in low temperature separation [2, 11]. Literature survey presents that optimum orifice diameter usually changes in the range of $0.4 < d_c/D < 0.6$ [51]. Westley [94] optimized experimentally a RHVT system and suggested the following equation for the ratio of cross section area of the cold orifice to the cross section area of the vortex tube:

$$\frac{A_c}{A_{vt}} \cong 0.167 \tag{21}$$

Table 2 presents optimum orifice diameter by Westley. Martynovski and Alekseev [15] determined that a decrease in orifice diameter results in an increase in cold temperature difference ($T_{in} - T_c$) until a critical value, then cold temperature difference decreases. Increasing orifice diameter causes to increase in hot temperature difference, but a decrease in efficiency of cold end results in. Takahama [20] reported that cold orifice diameter as a function of the nozzle diameter and nozzle number could be determined using the following equations:

$$d_c < D - 2d_n; \quad d_c^2/Nd_n^2 \leq 2.3 \tag{22}$$

Table 2 Optimum orifice diameters [11, 76]

d_n/D_h	d_c/D_h	
	$p_{in}/p_c = 1.5$	$p_{in}/p_c = 7$
0.266	0.34	0.38
0.376	0.37	0.38
0.461	0.40	0.40
0.532	0.41	0.37
0.595	0.36	0.32

Merkulov [103] suggested the following equation for dimensionless cold orifice diameter ratio.

$$\frac{d_c}{D} = 0.35 + 0.313\varepsilon \tag{23}$$

Soni [34] carried investigations with 170 different RHVT and suggested the following equation:

$$\frac{A_c}{A_{vt}} = 0.08 \approx 0.145 \tag{24}$$

Saidi and Valipour [81] investigated the optimum cold orifice diameter in a counter flow RHVT, as presented in Fig. 11. For $d_c < 0.5$, increasing d_c causes the efficiency to increase and for $d_c > 0.5$, the efficiency decreases. This indicates that the optimum value of d_c for the maximum cold air temperature difference and efficiency is 0.5 [81]. In 2005, Promvonge and Eiamsa-ard [61] observed that a small cold orifice ($d_c/D = 0.4$) yielded higher backpressure while a large cold orifice ($d_c/D = 0.7$, 0.8, and 0.9) allowed high tangential velocities into the cold tube, resulting in lower thermal/energy separation in the tube.

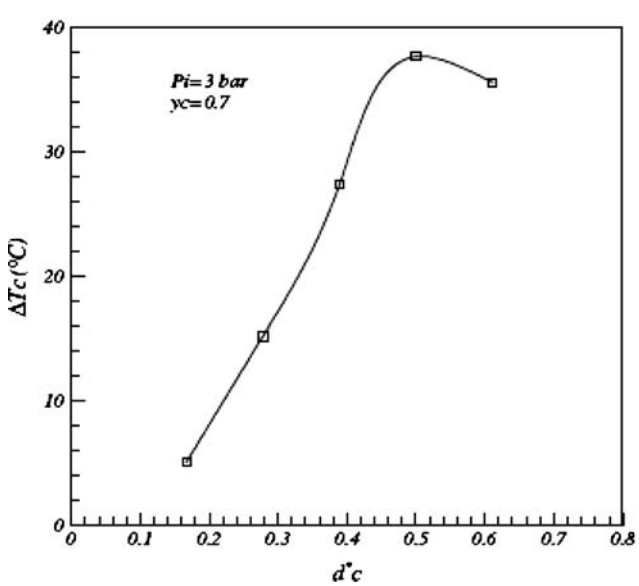

Fig. 11 Cold air temperature difference versus the dimensionless cold air orifice diameter [81]

The cold orifice diameter providing the highest temperature separation was 0.5. A CFD and experimental study have shown that for 12 mm diameter vortex tube, the cold end diameter of 7 mm is ideal for producing maximum hot gas temperature, while cold end diameter of 6 mm is optimum for reaching the minimum cold gas temperature [57]. These values correspond to 0.58 and 0.50 in L/D, respectively.

Martynovski and Alekseev [15] investigated different types of orifice and found that coaxial orifice was found to have greater temperature separation in compared to the other orifice configurations such as eccentric orifices, diaphram nozzles, and diaphrams with cross sections other than cylindrical configurations.

As a result it may be concluded that dimensionless cold air orifice diameter should be in the range of $0.4 < d_c/D < 0.6$ for optimum results.

4.6 Hot flow control valve

The volume and temperature of cold air produced by a RHVT are generally controlled by a control valve located in the hot exhaust. A conical valve at the end of this hot tube confines the exiting fluid to regions near the outer wall and restricts the gas in the central portion of the hot tube from making a direct exit. The conical valve is positioned to allow regulation of the relative quantities of hot and cold gases. By adjusting the valve, the cold fraction can be controlled. In general a cone-shaped valve is used in the RHVTs. In uni-flow RHVTs a cone-shaped valve with a central orifice is used. The cold air exit is located concentrically with the annular exit for the hot air. Blatt and Trusch [104] investigated experimentally the performance of a uniflow vortex tube and improved its performance by adding a radial diffuser to the end of the shortened tube instead of a cone valve. The flow field and the energy separation in a uni-flow vortex tube with an orifice rather than a conical valve to control the flow was experimentally investigated by Schlenz [105]. A conical valve made of mild steel was provided on the right hand side of the tube to regulate the flow by Singh et al. [56]. They studied the effect of the ratio of hot end area to the tube area (α) on the performance of a counterflow RHVT. α ranged from 0.07 to 0.27. The cold air temperature decreased with increasing α. Gao [1] experimentally investigated three different types of hot end plugs: spherical, plate-shaped and cone-shaped plug (Fig. 12). The parameters that characterized plug type were the number of the exhaust orifices, the diameter of the exhaust orifice, and the ratio of the total exhaust area and the vortex tube cross-sectional area in each type of plug. Energy separation effect was also observed in the experiments without the hot-end plug. It was concluded that the hot-end plug is not a critical component in the RHVT due to small differences between the results for different plugs.

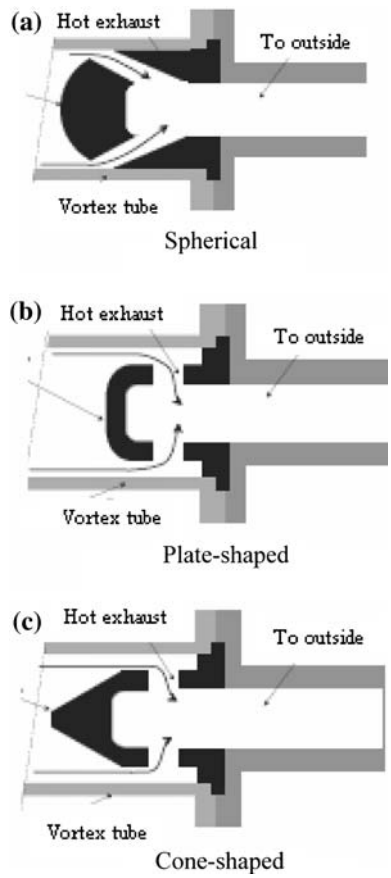

Fig. 12 The hot-end plugs used by Gao [1]

Aydın and Baki [83] investigated the effect of the angle of the cone-shaped control valve on the performance of a counterflow RHVT, changing the vane angle from 45° to 60°. The angle of the control valve yielding the optimum performance was 50°. Wu et al. [102] designed and installed a diffuser between the outlet of vortex tube and hot valve aiming to reducing the peripheral speed to zero within very short pipe and greatly reduce the ratio of length to diameter. It was determined that the cooling effect of the vortex tube with diffuser was up to 5°C lower than that without diffuser.

4.7 Tube geometry

Cylindrical geometry is usually used in RHVTs. Many investigations have been conducted in order to study the effect of tapering the RHVT. Martynovskii and Alekseev [15] found that a contracting and expanding vortex tubes had little effect on temperature separation; however, a contracting hot tube to be preferable. Otten [106] presented that the vortex tube performance could be improved by using a conical tube. It was found that an additional temperature drop of nearly 20°C at a cold mass flow ratio of 0.1, 10°C for a ratio of about 0.5, and almost no effect at ratios larger than 0.9. Parulekar [100] designed a short conical vortex tube. By varying the conical angle of the vortex tube, he found that the parameter L_{vt}/D_{vt} could be as small as 3. In 1964, Metenin [99] found that conical vortex tubes with a length of only $3D_h$ perform satisfactorily if a diffuser is added to the end of the hot tube. According to Gulyaev [78] a conical expansion brings improvements. The vortex tube with a conical angle of about 2.3° surpassed the best cylinder tube by 20–25% for the thermal efficiency and the refrigeration capacity. Borisenko et al. [31] concluded that a conical cold tube had no effect on a counterflow tube, while a 3° conical expansion in the hot tube resulted in optimal performance. Raiskii and Tunkel [33] in contrast found it is impossible to better performance of a cylindrical hot tube, noting that the majority of the energy transfer appears to occur within the first five diameters of the tube. In order to shorten the tube length, Takahama and Yokosawa [24] introduced the divergent (or conical) vortex tube. This divergent vortex tube could reach the same performance as the normal tube but with a smaller length. Because within the divergent tube the cross sectional area increases to the hot end, the gradient of the gas axial velocity decreases. He suggested that the divergence angle should be in the range 1.7°–5.1°. Gao [1] determined that rounding off the tube entrance increases the cold fraction in the RHVT system under the same operating conditions, and enhances the secondary circulation inside the RHVT itself and extends the circulation closer to the vortex chamber in the RHVT, at last improves the performance of the RHVT. The conical vortex tube was further investigated by Poshernev in 2003 and 2004 [53, 54] for chemical applications.

It is widely known that the RHVT system creates strong noise levels. In order to reduce the sound level and convert the acoustic energy into heat, in 1982 Kurosaka [37] and Chu [107], in 1983 Kuroda [108] introduced an acoustic muffler. They found that with the muffler the performance of the system was better than without.

The above investigations on the effect of tapering the RHVT present conflicting results. Although some investigators conclude that tapering the vortex tube for heating purposes have no effect on the performance, the others state that small tapering angles (~3°) leads to optimum results. With all research on divergent vortex tubes, it can be found that there exists an optimal conical angle and this angle is very small. This can be attributed slowing down the azimuthal motion of the flow. When the flow swirls to the hot end the cross section area increases, and so the azimuthal motion is slowed down along its path. Tapering the vortex tube contributes separation process in vortex tubes used for gas separation. Rounding off the tube entrance improves the performance of the RHVT [1, 24].

4.8 Cold flow mass ratio (cold fraction)

All investigators observed that cold and hot temperature changes significantly with cold fraction. In general, cold and hot temperature (or temperature differences) versus cold fraction is plotted in a graphic whose absicca axis represent cold fraction and ordinate axis cold and hot temperature (or temperature differences) (Fig. 13). The temperature of the cold exit air decreases with increasing cold fraction up to 0.3 and reaches there a minimum value. Then the temperature increases with further increases of the cold fraction higher than 0.3 [41, 51, 55, 109]. At very low air velocity, hence at cold fraction less than 0.3, the orifice plate causes the least loss and the temperature decreases with increases of pressure downstream the orifice plate. While at moderate and high air velocity by $\varepsilon > 0.3$ the orifice causes the highest loss. This is due to the more dominant influence of wall friction and due to the eddy zones. The static pressure drop across the orifice plate increases with higher cold fraction more than 0.3 due to the separation flow and eddy zones or back flow developed upstream and downstream the orifice. Following flow separation and abrupt change in the area of the flow, large-scale turbulence spreads through the flow; this produces a high-energy dissipation and a high friction loss, which may lead to a higher temperature [55].

The temperature of the hot exit air increases with increasing cold air mass ratio form null up to nearly 0.7–0.8. With further increases of the cold fraction higher than 0.7–0.8 the hot exit air temperature increases to a maximum value and then decreases to its minimum value. The range of the cold fraction, at which the hot exit air becomes a maximum, is from 0.8 to 0.82. At cold fraction from 0.82 up to one sharp temperature drop is measured for the hot air exit temperature. In fluid flow there are basically two causes of pressure variation in addition to the weight effect- these are acceleration and viscous resistance. In the range of cold fraction ratio of 1 up to 0.82, hence relative increases of the hot mass flow rate, the acceleration in the inner pipe is not enough large to overcomes the viscous resistance. The friction between the air molecular and the friction between the air and the pipe wall is very high so that the temperature of the hot exit air increases and reaches its highest value. At cold fraction lower than 0.82 more air flows into the hot outlet pipe. The acceleration is then enough large to overcome the viscous resistance, hence the friction. A decrease in the cold air mass ratio from 0.82 up to 0 causes a decrease in the wall friction effect; temperature drop as a consequence [55].

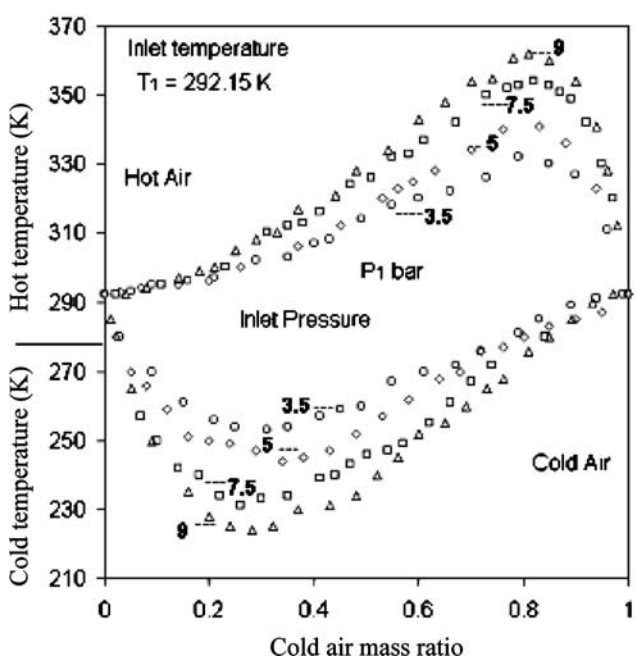

Fig. 13 Hot and cold exit temperatures as a function of the cold air mass ratio and the inlet pressure [55]

By studying the variation of exergy destruction with cold air mass flow ratio, Saidi and Yazdi [50] determined that minimum exergy destruction occurs at about $\varepsilon = 0.7$. This means that the efficient working point of the vortex tube is at $\varepsilon = 0.7$. Cold fraction as well as adiabatic efficiency is more influenced by the size of the cold orifice rather than the size of the nozzle [56]. Gao [1] determined that the flow pattern in the RHVT shows secondary circulation inside the vortex tube. The flow pattern depends on the type of the working fluid, the normalised pressure ratio, and the cold fraction. Aydın and Baki [83] showed that the cold fraction is an important parameter influencing the performance of the energy separation in the RHVT. The effect of nozzle design is more important than the cold orifice design in getting higher temperature drops [56]. Linderstrom-Lang [27] studied in detail the application of the vortex tube to gas separation, using different gas mixtures and tube geometry and found that the separation effect depended mainly on the ratio of cold and hot gas mass flow rates.

Commercial vortex tube manufacturers produce two types of vortex tube design. They are called as “maximum refrigeration design” and “minimum temperature design”. Higher temperature drops are obtained in vortex tube made of maximum temperature drop tube design, whereas, more cold fraction and higher adiabatic efficiency are obtained with maximum cooling effect tube design. Maximum refrigeration occurs when a RHVT operates at 60–70% cold fraction. This is where the product of the mass of cold air and its temperature drop is the greatest. Cooling machining operations, electronic controls, liquid baths, and workers will require high cold fractions for maximum refrigeration. These applications use this maximum refrigeration setting. On the other hand, some applications

require the lowest possible cold output temperature. Colder temperatures are useful for cooling glass, cooling hot parts, laboratory experiments, and testing of electronic components. A low cold fraction (i.e. less than 50% of the input air exiting through the cold air exhaust), produces the lowest temperatures, but with reduced airflow [84–93].

4.9 Inlet pressure and temperature of gas

Martynovskii and Alekseev [15] reported that increasing the inlet pressure resulted in large temperature difference between hot and cold gas temperatures. In 1957, Westley found that the reduced maximum temperature is a function of reduced inlet pressure and it approaches an asymptotic value as the reduced inlet pressure increases. This asymptotic value approximately equals to a value of 0.22 for optimum cold gas flow, the optimum inlet nozzle and the cold gas orifice diameter. For an inlet gas at 300 K, this corresponds to a maximum temperature drop of about 66 K. Hilsch [5] determined that the temperature drop was 68 K with an inlet pressure of 11. Westley also reported that the maximum temperature drop increases very slightly beyond the value of p_{in}/p_c of 11 or 12. Lewellen [79] conducted a very simplified analysis in which he predicted an asymptotic value of about 0.185. Although this value is somewhat lower than Westley's asymptotic value of 0.22, Lewellen reports that "it is close enough to suggest that the model is basically sound." Gulyaev [77] investigated vortex tube performance using Helyum at temperatures of 80 K. He found that the cold gas temperature is approximately equal to a constant times the inlet temperature of gas

$$\frac{T_{in} - T_c}{T_{in}} = \text{constant} \quad (\text{for a given } p_{in}/p_c \text{ and } \varepsilon) \tag{25}$$

At the optimum conditions the above constant should be the same as the asymptotic value mentioned above. According to Gulyaev the ratio, $(T_{in} - T_c)/(T_{in} - T_c)_{is}$, varies in the range of 0.2–0.5. The ratio equals to a value of about 0.2 at a cold gas flow ratio of 0.8, to about 0.5 at cold gas flow ratios around 0.2–0.3. Rather than depending on the absolute inlet pressure, the temperature separation was found to be a linear function of the normalised pressure drop $(X = (p_{in} - p_c)/p_{in})$ between the inlet and the cold end of the vortex tube [43]. Ahlborn et al. [43] also found that vortex tubes behave identically in the above and below atmospheric pressure regimes. Essential for the temperature splitting is the normalised pressure drop X and not the absolute entrance pressure. They concluded that safe incorporation of vortex tubes in closed cycles that may go below atmospheric pressures for cooling and dehydration applications. Temperature difference increases and exergy destruction decreases as inlet pressure increases [50]. According to Saidi and Valipour [81] the cold air temperature difference increases by increasing the inlet pressure, meanwhile there is an optimum efficiency at specific inlet pressure. Shannak [55] found that a 40% change of the inlet pressure may lead to 2% change of the hot and cold temperature. Maximum irreversibility factor is a function of the ratio of inlet pressure to the cold flow pressure, (p_{in}/p_c) [1]. Aydın and Baki [83] experimentally observed in a counterflow RHVT that the higher the inlet pressure, the greater the temperature difference of the outlet streams.

According to Martynovskii and Alekseev [15] inlet temperature does not affect significantly the stagnation temperature differences. Gulyaev [77] reported that the ratio of $(T_g - T_c)/(T_g - T_c)_{is}$ was nearly independent of the inlet gas temperature. Another investigation showed that a 2% change of inlet temperature could lead to change the temperature of hot and cold exit air up to 2% [55]. Ma et al. [110] observed experimentally that the hot stream temperature and cold stream temperature increase slightly with inlet temperature rising, whereas the temperature difference between the hot and the cold air stream decreases slightly with inlet temperature rising. They concluded that the inlet temperature is not a strong factor influencing the energy separation just in their experimental range.

As a result it may be concluded that although the inlet temperature has negligible effect on performance the increase of inlet pressure enhances temperature separation.

4.10 Type of fluid

Air is generally used in vortex tubes and quite low temperature drop could be obtained. There are also investigations with working fluids such as steam, hydrocarbons and other gases.

4.10.1 Various gases

Based on the governing equations a similarity relation, the ratio of the actual temperature drop of the cold gas to the maximum temperature drop, $(\Delta T_c/\Delta T_{cmax})$, for geometrically similar vortex tubes was established by Stephan et al. [39]. This similarity relation was found to be independent from the operating conditions and the working substances in vortex tubes that are geometrically similar. The experiments conducted with air, helium and oxygen as working media confirmed the similarity relation. Heating-cooling temperatures of air, oxygen, carbon dioxide and nitrogen as working fluid in vortex tubes have been investigated experimentally [111]. Oxygen, carbon dioxide and nitrogen provided lower cold temperatures than air. Carbon dioxide was found to have lower hot and cold flow temperatures in

compared the other fluids. Gao et al. [1] built a simple vortex tube and used nitrogen as its working fluid. In 2006, Aydın and Baki [83] obtained that nitrogen provides higher temperature differences in compared to air and oxygen. The reason of this behaviour was attributed to the molecular weight of nitrogen, which is much smaller than that of air and oxygen.

4.10.2 Gaseous hydrocarbons

Gaseous hydrocarbons also exhibit the Ranque–Hilsch effect, as presented by a number of investigators. In 1971, Williams [112] used methane in a counterflow RHVT, obtaining performance characteristics very similar to those of air. The measurements of Collins and Lovelace [113] with a two-phase, liquid–vapour mixture, propane in a standard counter-flow vortex tube showed that for an inlet pressure of 0.791 MPa, the separation remained significant for a dryness fraction above 80% at the inlet. With a dryness fraction below 80%, the temperature separation became insignificant. But the discharge enthalpies showed considerable differences indicating that the Ranque–Hilsch process is still in effect. Ahlborn et al. [45] designed vortex tubes with R22, R134 ve R144 as working fluids.

4.10.3 Steam

Starostin and Itkin [114] reported that in order to obtain fundamental temperature separation, greater inlet pressure is required in steam than air. Performance of vortex tube with steam was found to be similar to that of air. Some criteria and expressions to estimate the energy separation performance for steam operated vortex tube were introduced by Takahama et al. [23]. Study by Takahama et al. reveals that (1) geometrical dimensions for vortex tube operated with air can be used for one operated with steam. (2) As far as steam is in the superheated region at the inlet nozzle, the energy separation performance is the same as that for air and is presented by the same curve independently of the degree of superheat, total mass flow rate and discharge resistance. (3) When steam is in the wet region at the nozzle outlet, even though steam supplied is superheated, the performance considerably decreases because of the energy waste from moisture vaporization. No energy is separated when the dryness fraction at the nozzle outlet is less than approximately 0.98.

4.10.4 Water

In 1988, Balmer [40] introduced high pressure water as the working medium into a commercially available counterflow RHVT. He showed that there is no thermodynamic reason why energy separation in liquids should not occur, and found experimental evidence of a radial temperature gradient at very high inlet pressure. It was found that when the inlet pressure is high, for instance 20–50 bar, the energy separation effect still exists. So it proves that the energy separation process exists in incompressible vortex flow as well.

In general it may be stated that thermophysical characteristics of fluids influence the performance of vortex tubes, performance declines with increasing wetness fractions, and performance characteristics for vortex tubes with steam and hydrocarbons are very similar to those of air.

4.11 Gas properties

Otten [106] suggested a correction factor for $T_{in} - T_c$ as a function of c_p/c_v (k). The ratio of the specific heats is used in connection with a plot giving the standard performance of a vortex tube operating with air. According to Otten if the ratio of specific heats varies with pressure or if the Joule–Thompson effect is important, appropriate gas table should be used. Temperature drops for various gases are presented in Table 3 [15]. In this table cold mass flow ratio is 0.3 and reduced pressure ratio (p_{in}/p_c) is 5.0. The lower temperature drop of air than that of methane may be attributed to moisture content of air. Maximum temperature drop is proportional to Prandtl number which is one of the most important properties of gas type. Specific heat ratio (k) is the inlet gas characteristic that affects the amount of energy separation in the vortex tube. Cold temperature difference increases with increasing k [56, 81]. Piralishvili and Fuzeeva [71] finds that the values of the relative cooling of different gases lie near one straight line described by the equation

$$\frac{\Delta T}{\Delta T_{air}} = 1.51 - 0.295\left(\frac{kT_{cr}}{\varepsilon_0}\right) \tag{26}$$

where k is the Boltzmann constant (J/K), T_{cr} is the critical temperature, and ε_o is the Lennard–Jones potential (the minimum potential energy of the intermolecular interaction).

When the inlet gas contains moisture the energy separation and tube efficiency considerably decreases [15]. Saidi and Valipour [81] injected water into the inlet flow in

Table 3 Temperature drop variation for various gas [11, 15]

Gas	Temperature drop ($T_{in} - T_c$)	Prandtl number	Heat capacity ratio
Air	38.0	0.73	1.403
CH_4	40.0		1.310
CO_2	34.6	0.78	1.304
NH_3	30.0	0.85	1.310

Table 4 Important common results on design criteria of vortex tubes

Group name	Parameter/variable name	Important common results
Geometrical parameters	Tube length	The length of the vortex tube affects performance significantly.
		An efficient tube of either design should be many times longer than its diameter.
		Optimum L/D is a function of geometrical and operating parameters.
		The magnitude of the energy separation increases as the length of the vortex tube increases to a critical length, however a further increase of the vortex tube length beyond the critical length does not improve the energy separation.
		L/D has no effect on performance beyond $L/D > 45$.
	Tube diameter	Different vortex tube diameters have been used in experimental RHVT investigations, from diameters as low as 4.4 mm and as high as 800 mm.
		The vortex tubes used for gas liquefication and separation can have much greater diameters.
		In general smaller diameter vortex tubes provide more temperature separation than larger diameter ones.
		A very small diameter vortex tube leads to low diffusion of kinetic energy which also means low temperature separation.
		A very large tube diameter would result in lower overall tangential velocities both in the core and in the periphery region that would produce low diffusion of mean kinetic energy and also low temperature.
	Diameter/area of inlet nozzles	Increasing nozzle diameter, generally, increases the performance.
		The optimum nozzle diameter is about to be 0.25 D_{vt}.
	Type and number of nozzles	For maximum temperature drop the inlet nozzles should be designed so that the flow be tangentially into vortex tube.
		The increase of the number of inlet nozzles leads to higher temperature separation.
		The inlet nozzle location should be as close as possible to the orifice to yield high tangential velocities near the orifice.
	Cold orifice	Using a small cold orifice ($d_c/D = 0.2$, 0.3, and 0.4) yields higher backpressure while a large cold orifice ($d_c/D = 0.6$, 0.7, 0.8, and 0.9) allows high tangential velocities into the cold tube, resulting in lower thermal/energy separation in the tube.
		Dimensionless cold orifice diameter should be in the range of $0.4 < d_c/D < 0.6$ for optimum results.
		Coaxial orifices have greater temperature separation in compared to the other orifice configurations such as eccentric orifices, diaphram nozzles, and diaphrams with cross sections other than cylindrical configurations.
	Hot flow control valve	The hot-end plug is not a critical component in the RHVT.
		Optimum value for the angle of the cone-shaped control valve (ϕ) is approximately 50°.
	Tube geometry	Tapering the vortex tube contributes separation process in vortex tubes used for gas separation.
		In divergent vortex tubes, there exists an optimal conical angle and this angle is very small (~ 3).
		Rounding off the tube entrance improves the performance of the RHVT.
		With the muffler the performance of the system is better than that of without muffler.
Mass flows	Cold mass fraction	The cold and hot temperature changes significantly with cold fraction.
		Higher temperature drops are obtained in vortex tube made of minimum cold flow temperature design, whereas, more cold fraction and higher adiabatic efficiency are obtained with maximum cooling capacity design.
		Maximum refrigeration occurs when a RHVT operates at 60–70% cold fraction.
		Minimum cold temperature occurs when a RHVT operates at 30 percent cold fraction.
	Overall mass flow rate	The inlet or overall mass flow rate of the working gas supplied into the vortex tube is one of the important factors affecting the performance.

Table 4 continued

Group name	Parameter/variable name	Important common results
Reservoir conditions	Inlet (reservoir) pressure	The increase of inlet pressure enhances temperature separation.
		Rather than depending on the absolute inlet pressure, the temperature separation is a linear function of the normalised pressure drop between the inlet and the cold end of the vortex tube.
		The normalised maximum temperature is a function of normalised inlet pressure and it approaches an asymptotic value as the normalised inlet pressure increases.
		The maximum temperature drop increases very slightly beyond the value of $p_{in}/pc = 11$ or 12.
		Vortex tubes behave identically in the above and below atmospheric pressure regimes.
	Inlet (reservoir) temperature	Inlet temperature does not affect significantly the temperature differences and performance.
Gas properties	Gas Prandtl number	Maximum temperature drop is proportional to Prandtl number.
	Gas isentropic exponent	Specific heat ratio (k) is the inlet gas characteristic that affects the amount of energy separation in the vortex tube.
		Cold temperature difference increases with increasing k.
	Moisture content	The cold temperature difference and efficiency decrease by increasing the air moisture content of air.
Other factors	Tube material	Using materials with more smooth surfaces and lower thermal conductivities results in better temperature separation and performance.
		Using the vortex tube with insulation to reduce energy loss to surroundings gives a higher temperature separation in the tube than that without insulation.
		For all feasible operations of the vortex tube, choice of a durable material for the manufacture of the tube is quite important.
	Internal roughness	The roughness of the inner surface of the tube has influence on its performance.
		Any roughness element on the inner surface of tube will decrease the performance of the system.
		Using materials with more smooth surfaces results in better temperature separation and performance.
	Gas molecular mass	The lighter the molecular weight, the higher the temperature separation.
		Inlet gas with helium gives higher temperature difference than those found from the oxygen, methane, and air.
		Performance of RHVT with steam and hydrocarbons is similar to that of air.
		When the inlet pressure is high the energy separation process exists in incompressible vortex flow. For water very high pressures, 20–50 MPa, are required.

order to determine the effect of air moisture on energy separation. The cold temperature difference and efficiency decreased by increasing the air moisture content of air.

4.12 Tube material

Stainless steel is usually used as tube material in vortex tubes; however, other materials have also been used, too. These materials may be grouped into two general classes: metal and plastic materials. Metal materials include steel, copper, aluminium, alloys etc., plastic materials perspex, capralon, polystyren etc.

There have been some investigations studying the effect of using different material on the performance of vortex tubes. According to Parulekar [100] the roughness of the inner surface of the tube has influence on its performance as well: any roughness element on the inner surface of tube will decrease the performance of the system (based on the temperature difference) up to 20%. Saidi and Yazdi [50] observed that steel tube gives higher temperature differences ($T_h - T_c$) than that of PVC tube. They concluded that using materials with more smooth surfaces and lower thermal conductivities results in better second law efficiency. According to Singh [75], the performance of the perspex tube is generally better than that of the brass tube. The generally lower efficiency of the brass tube could be due to its better conductivity as compared to the perspex tube. On the other hand perspex could be too a fragile material to withstand the very high pressure of the inlet air. For these reasons, the product life of the brass tube should

be longer. Using the tube with insulation to reduce energy loss to surroundings gave a higher temperature separation in the tube than that without insulation around 2–3°C for the cold tube and 2–5°C for the hot tube [61]. Azarov [64–68] developed many projects on vortex tubes for various industrial applications and manufactured various vortex tubes with different designs. Stainless steel, brass, aluminium, aluminium alloys, capralon, textolite etc. have been used in these vortex tubes.

All investigators concluded that using materials with more smooth surfaces and lower thermal conductivities, and using the tube with insulation to reduce energy loss to surroundings results in better temperature separation and performance. It is important to note that for all feasible operations of the vortex tube, choice of a durable material for the manufacture of the tube is also quite important [115, 116].

5 Conclusions

Vortex tube is a device without moving mechanical parts, which converts a gas flow initially homogeneous in temperature, into two separate flows of differing temperatures. Since its discovery in 1930 by Ranque, the vortex tube has been the subject of considerable interest both from the theoretical and practical application standpoints. However, it is difficult to design a vortex tube with definite integral characteristics for a concrete application because the available experimental data are not clearly understood and there are no entirely correct generalizations. The purpose of this article is to overview of the past investigations of the design criteria of vortex tubes, to draw together the mass of literature, and to provide detailed information on the design of vortex tubes. Thus it will be possible to access the results of available experimental/theoretical investigations on vortex tubes. It will be possible also to make generalization about design of vortex tubes. First the classification of vortex tubes is presented and the types of vortex tubes are described. Then all criteria on the design of vortex tubes are given in detail using experimental and theoretical results from the past until now. Finally the criteria on the design of vortex tubes are summarized. Important common findings deduced from the literature survey are tabulated and summarized in Table 4.

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