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Vortex tube optimization theory

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Abstract

The Ranque–Hilsch vortex tube splits a single high pressure stream of gas into cold and warm streams. Simple models for the vortex tube combined with regenerative precooling are given from which an optimization can be undertaken. Two such optimizations are needed: the first shows that at any given cut or fraction of the cold stream, the best refrigerative load, allowing for the temperature lift, is nearly half the maximum loading that would result in no lift. The second optimization shows that the optimum cut is an equal division of the vortex streams between hot and cold. Bounds are obtainable within this theory for the performance of the system for a given gas and pressure ratio.

1. Introduction

The Ranque–Hilsch vortex tube [1–3] is an intriguing device in which a stream of high pressure gas swirls to split into two low pressure streams, one higher than at entry temperature and the other lower. The drop in pressure more than accounts for this apparent breach of the Second Law of Thermodynamics, which nevertheless puts a limit on performance. At a pressure drop ratio of 5 and ambient temperature, temperature differences of 30 K (or 10% of ambient) are easily obtained, sufficient for simple refrigeration. The lack of moving parts, electricity etc., make the device attractive for a number of specialized applications where simplicity, robustness, reliability and general safety are desired, either as a supply of hot or, more likely, cold gas. One thinks of warming deep sea divers and cooling firemen where the only fault anticipated is the loss of air supply, in which case the user has other things to worry about.

The performance of the tube, and hence any optimum design, varies with the ‘cut’ or the

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Nomenclature

A	heat exchange area, m^2
c_p	specific heat capacity, J/kg
k	effective conductivity, W/m
L	active tube length, m
$\dot{m}$	mass flow rate into tube, kg/s
M	tube size, dimensionless
N	smaller flow-side tube size, dimensionless
NTU	nondimensional heat transfer units
p	pressure, bar
q	refrigeration $\dot{Q}/\dot{m}c_p$
$\dot{Q}$	refrigerative heat flow rate, W
r	pressure ratio, inlet-outlet
r_0	effective tube radius, m
R	ideal gas constant, J/kg K
s	specific entropy, J/kg K
$\dot{S}$	entropy rate, W/K
T	temperature, K
$\dot{W}$	power, W
x	cut, or cold mass flow fraction

Greek symbols

γ	ratio of specific heat capacities, c_p/c_v
Γ	pressure ratio parameter, $(p_i/p_0)^{(\gamma-1)/\gamma}$
ΔT	$T_H - T_C$
ΔT_C	$T_0 - T_C$
ΔT_H	$T_H - T_0$
$\Delta\theta$	internal temperature difference, K
ϵ	heat exchanger effectiveness

Subscripts

C	cold
gen	generated
H	hot
i	inlet
opt1	first-optimized
opt1,2	first- and second-optimized
p	constant pressure
Q	refrigeration
0	ambient

fraction of the stream diverted to the cold side. In any particular application one would need to know in detail the performance characteristics. These are not easily measured since the tendency is to measure the temperatures at some part of the tube rather than the gas temperatures. Furthermore, the gas flows themselves are not easily measured accurately. Hence, for a general discussion it would be useful to have an elementary model of how performance varied with the cut. This is our purpose here, focusing on the unique cooling effect. We suppose that it is desired to maximize useful refrigeration; this can be done in part by using the cold stream to pre-cool the inlet stream. Optimization is called for to assess what refrigeration load to place on a tube in given performance and secondly to optimize the performance or cut, the proportion of cold exit stream at a given ambient temperature and pressure supply ratio.

The assessment starts as a First Law balance. The Second Law then is invoked (firstly as simple Carnot theory and then as a minimization of entropy production) to obtain the best refrigeration effect of given low conditions. To provide the second optimization, over the variable cut, involves us in a model for the vortex tube as a heat exchanger between two internal streams. We have developed such a model from order-of-magnitude or scaling considerations which may therefore be helpful to potential users.

The literature on the Ranque–Hilsch tube is extensive. Hartnett and Eckert [4] and Eckert [5] are perhaps typical of experimental studies in air. Effects in steam and other gases are reported in Starostin and Itkin [6], and Williams [7]. The heat-exchanger analogy can be attributed to Scheper [8] and is developed by Linderstrom–Lang [9]. A useful comprehensive study is that of Gulyaev [10]. Our development of the heat-exchanger analogy in this paper is notable for supposing that an inefficient exchanger is conducive to a good temperature difference and hence limits the length-to-diameter ratio, somewhat in contrast with Hilsch's own comment [2].

2. First law balance

Fig. 1 shows a schematic tube in which entry gas at inlet temperature T_I is equal to ambient temperature T_0, and inlet pressure p_I swirls along the exchanger portion of the tube. The swirl establishes a radial pressure difference leading to radial changes of specific volume; these changes diminish along the active length of the tube L. A fraction $1-x$ of the mass flow rate escapes through the controlled annulus to the left at T_H, and the remaining fraction x returns to pass through a diaphragm, exiting to the right at T_C. The gases exit at atmospheric pressure p_0. It is generally found that the outer gases are hotter, exiting to the left, and the inner gases colder, exiting right.

In the pressure ranges often encountered, principally in air, the expanding gas can be treated as ideal and hence with no Joule–Thomson heating or cooling effect. A simple explanation is then that the inner core expands and does work (either as expansion or as shear work due to relative tangential velocity) on the outer annulus and thus cools, while heating the outer annulus. During this process there will of course be an exchange of heat between the fluids at different temperatures, offsetting the vortex effect. A more exact account is a very considerable challenge to computational fluid dynamics and will not be attempted here. We assume a perfect gas obeying ideal gas laws and having constant specific heat capacity c_p.

Fig. 2 gives the schematic for the temperatures entering and leaving the tube. Assuming a well-insulated tube, the First Law requires

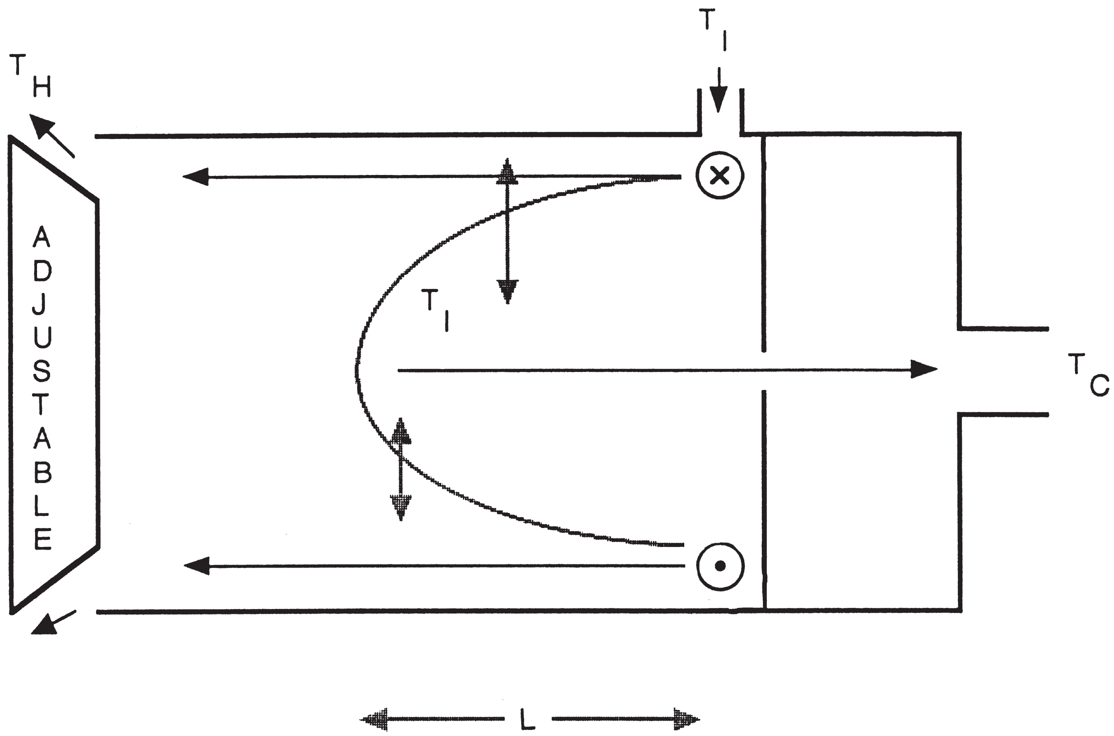

Fig. 1. The vortex tube as a counterflow heat exchanger.

$$\dot{m}c_pT_0=\dot{m}c_p(1-x)T_H+\dot{m}c_pxT_C \quad (1)$$

so that writing ΔT for the total temperature effect between the tube exits, ΔT_C for the cooling effect (entry to cold exit), and ΔT_H for the heating effect,

$$\Delta T_C=(1-x)\Delta T, \quad \Delta T_H=x\Delta T. \quad (2)$$

Correspondingly, $\Delta T = \Delta T_H + \Delta T_C$. In practice Eq. (2) may well be used in reverse, to determine the cut from the measured temperatures, if these are realistically measurable.

The cooling effect can be used two ways. Firstly, it might simply be used to take heat from some body at a temperature T_Q, say, extracting at a rate $\dot{Q}$, and raising the cold temperature accordingly; if the rise in temperature is fixed, so is the refrigeration effect. Secondly, the cold stream might be used in a heat exchanger to cool the incoming stream itself, in a regenerative fashion, thus lowering the exit temperature still further. Obviously these two effects compete; maximum refrigeration denies regenerative cooling, and maximum regenerative cooling denies refrigeration.

We assume for simplicity that the regenerative heat exchanger (Fig. 2) has an effectiveness of unity in what follows, thus bringing the return temperature up to ambient. Given that the streams are of different capacities for general x, and the temperature differences along the heat exchange are therefore not constant, this is not a bad assumption; the mean temperature difference between

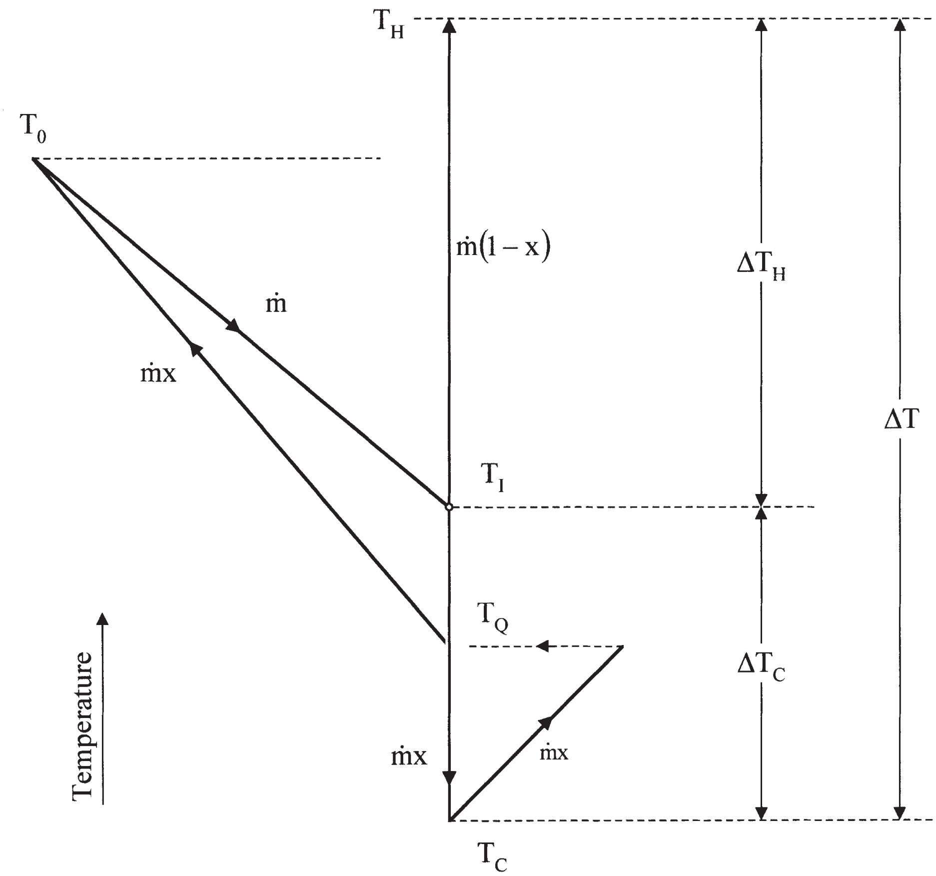

Fig. 2. The vortex tube with regenerative heat exchanger with refrigerative section.

the streams is finite even if it is assumed to vanish at one end.[1] We also assume that the lowering of the tube inlet temperature below ambient by regenerative cooling does not affect the temperature differences induced within the tube itself, although these of course do depend upon the cut. Next we suppose that the refrigeration load is again applied through a heat exchanger of effectiveness unity so that the heat to be pumped enters the system at the temperature to which the cold stream has been raised.

We can now undertake an enthalpy balance for the combination of vortex tube and regenerative heat exchanger admitting a quantity of heat $\dot{Q}$ supplied from the constant temperature level T_Q. Putting $q = \dot{Q}/\dot{m}c_p$, where q is the temperature change that would be achieved in the entry stream, we have

$$T_0-T_Q=\Delta T-q/[x(1-x)]. \tag{3}$$

[1] 100% effectiveness requires an infinite exchanger but an effectiveness of 95% is achieved for a cut of one half by an exchanger of less than ten NTU.

When the maximum heat is pumped, the left-hand side is zero and there will be no refrigeration gain. This maximum is $q_{max} = x(1-x)\Delta T$. On the other hand, the lowest refrigeration temperature $T_Q=T_C$ arises when no heat is pumped.

The refrigeration effect will be in part quantitative (namely, the amount of heat that can be pumped up to ambient temperature) and part qualitative, depending on the temperature lift given. Therefore at any given ‘cut’ there is a first optimization to be found. One way to assess this is to characterize the quality of the heat pumped in terms of the minimum (or reversible) work that would have to be done over the temperature interval. That is, the equivalent reversible power needed is $\dot{W} = \dot{Q}(T_0/T_Q-1)$, or nondimensionally

$$\frac{\dot{W}}{\dot{m}c_p T_0}=\frac{q}{T_0}\left(\left[\frac{\Delta T}{T_0}-\frac{q}{x(1-x)T_0}\right]\Big/\left[1+\frac{q}{x(1-x)T_0}-\frac{\Delta T}{T_0}\right]\right). \tag{4}$$

At fixed cut, this can be optimized over q, yielding

$$\frac{q_{opt}}{T_0}=x(1-x)\left(1-\frac{\Delta T}{T_0}\right)^{1/2}\left[1-\left(1-\frac{\Delta T}{T_0}\right)^{1/2}\right]\cong x(1-x)\frac{\Delta T}{2T_0}\left(1-\frac{\Delta T}{4T_0}\right). \tag{5}$$

It can be shown that the extremum represented by Eq. (5) is a maximum. To first order in $\Delta T/T_0$ this result guides us to half the maximum cooling heat transfer, developing half the maximum refrigeration temperature drop. The second-order correction is worth only 2×(1/2)% when $\Delta T/T_0$ is 10% and hence no higher-order terms are considered in the expansion. For use later, we note that the exact Eq. (5) has a maximum over the temperature difference when $\Delta T/T_0 \rightarrow 3/4$ although the tube will generally operate well below such a performance.

Note that this approach gives no credit to the work potential of the hot exit stream; the size of temperature intervals available would make such exploitation impractical. More significantly, the ‘credit’ wanted is the minimum power to raise the heat to ambient, not above. In Eq. (4) the non-dimensional value factor of a unit of heat pumped from temperature T_Q up to T_0 is $[(T_0/T_Q)-1]$; this is the minimum (reversible, Carnot) work to achieve this refrigeration effect. It discounts the actually greater temperature at which the hot air is discharged as being unavailable and a matter of external entropy generation. Equally, it discounts anything lower than the T_Q temperature achieved in the process of extracting heat from the cold source at such a temperature, since this is not of interest to the purchaser of refrigeration, again a matter of external entropy generation.

We may also approach this first optimization as one of minimizing the entropy generation rate. To do this it is essential to include the entropy generated everywhere, including the exiting hot stream [11]. We have

$$\text{internal: } \dot{S}_{gen}=\dot{m}[xs(p_0,T_0)+(1-x)s(p_0,T_H)-s(p_I,T_0)]-\dot{Q}/T_{\dot{Q}}.$$

Substituting perfect gas expressions this becomes

$$\text{internal: } \frac{\dot{S}_{gen}}{\dot{m}c_p}=\frac{R}{c_p}\ln\frac{p_I}{p_0}-\frac{q}{T_Q}+(1-x)\ln\frac{T_H}{T_0}. \tag{6}$$

But the hot stream exits wastefully and cools to ambient generating more entropy as

$$\text{external: } \frac{\dot{S}_{\text{gen}}}{\dot{m}c_{\text{p}}}=(1-x)\int_{T_{\text{H}}}^{T_0}\left(\frac{1}{T}-\frac{1}{T_0}\right)\mathrm{d}T=-(1-x)\left(\ln\frac{T_{\text{H}}}{T_0}-\frac{T_{\text{H}}-T_0}{T_0}\right). \tag{7}$$

In combination therefore

$$\text{total: } \frac{\dot{S}_{\text{gen}}}{\dot{m}c_{\text{p}}}=\frac{R}{c_{\text{p}}}\ln\frac{p_{\text{I}}}{p_0}-\frac{q}{T_{\text{Q}}}+(1-x)\frac{T_{\text{H}}-T_0}{T_0}. \tag{8}$$

When this quantity is minimized over the refrigeration load (the pressure ratio unchanged) we have the same result as from Eq. (4), since $\dot{Q}=(1-x)\dot{m}c_{\text{p}}(T_{\text{H}}-T_0)$.

3. The vortex tube as a heat exchanger

We seek a model that will tell us how the temperature difference between hot and hold streams, ΔT, will vary with the cut. We provide this in an order of magnitude sense [12] consistent with the other simplifying assumptions. We can suppose that the gas expands in the vortex and can produce an internal temperature difference $\Delta\theta$. This is not the difference between the exiting gas streams but the radial difference, the maximum internal temperature difference that can be produced. Before losses, this will satisfy (in an order of magnitude sense) the reversible thermodynamics relation for an adiabatic change, i.e.

$$\ln\left(1+\frac{\Delta\theta}{T_0}\right)=\frac{R}{c_{\text{p}}}\ln\left(\frac{p_{\text{I}}}{p_0}\right),\text{ where }\frac{R}{c_{\text{p}}}=\frac{\gamma-1}{\gamma}. \tag{9}$$

The losses will be two-fold in nature. First the expansion, involving turbulent flow, will not be reversible and the maximum tube temperature difference will be modified. Secondly, there will be heat exchange between the two streams formed at different temperatures; this exchange will be either by thermal diffusion or by its equivalent in turbulent exchange. We model only the second of these irreversibilities and note that we would wish the vortex tube to be a *poor* heat exchanger.

Return to Fig. 1 which shows the tube as a counterflow heat exchanger. We may suppose there exists within the tube some surface at inlet temperature dividing the hot from the cold stream; turbulent exchange takes place through this surface. There is then an active length L along which heat is exchanged with an effective thermal conductivity k. If the radial length scale is r_0, then conventional heat exchange theory [13, pp. 468–472] may be applied. The UA value of the tube as a heat exchanger is $2\pi(k/r_0)r_0L$, and we can write a modified nondimensional heat exchanger size, in NTU units as $M=2\pi kL/(\dot{m}c_{\text{p}})$, notably independent of the radial size.

The expression for the effectiveness of a heat exchanger in counterflow with unbalanced streams, as in the present case, is [13, pp. 468–472]: $\varepsilon = \{1 - \exp[-N(1-r)]\}/\{1-r\exp[-N(1-r)]\}$, where the nondimensional size N is to be based on the stream with the smaller capacity. The ratio of the capacities is $r = \dot{m}c_{p,\mathrm{smaller}}/\dot{m}c_{p,\mathrm{bigger}} \leq 1$. Hence $N = M/x$, $x<1/2$ and $N = M/(1-x)$, $x>1/2$. The effectiveness must be treated carefully for the case of equal streams, or found from first principles: $\varepsilon_{\mathrm{balanced}}=2M/(1+2M)$. In the present application the cut leads to the following:

$$\varepsilon=\{1-\exp[-(M/x)(1-2x)/(1-x)]\}/\{1-[x/(1-x)]\exp[-(M/x)(1-2x)/(1-x)]\},\ \text{when } x>1/2. \tag{10}$$

We now view the exchange process as having incoming streams separated by the tube characteristic temperature $\Delta\theta$ while exiting at the gas temperature difference ΔT, itself depending on the effectiveness and hence the cut. Fig. 3 shows schematically the temperature changes in this heat exchanger. From the definition of effectiveness — the actual temperature change in the stream of smaller capacity divided by the maximum feasible temperature change — we obtain

$$\Delta T=\Delta\theta[1-\varepsilon/(1-x)],\ x<1/2 \text{ and } \Delta T=\Delta\theta(1-\varepsilon/x),\ x>1/2. \tag{11}$$

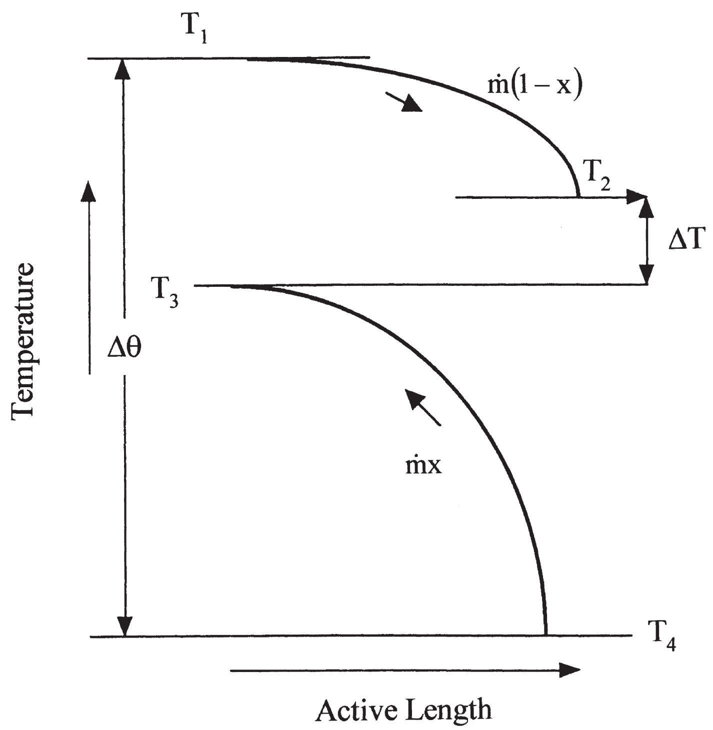

Fig. 3. Temperature distributions along the vortex tube.

This relationship is plotted in Fig. 4 for a range of modified heat exchanger sizes M. It shows that only for small M (less than 0.5, compact exchangers, therefore of short working length[2] L), does the tube provide significant heating and cooling effect at the exits, however big the characteristic change $\Delta\theta$ might be. No physical meaning is given here to the curves falling below zero which have been correspondingly terminated in the figure.[3]

Note that ΔT peaks at $x = 1/2$ for any cut. This may be demonstrated analytically by expanding about the mid-point and observing that the function is continuous, with zero slope, and exhibits a maximum. Writing $y = x-1/2$, we have

$$\frac{\varepsilon}{1-x}=\left[1-\exp\left(\frac{8yM}{1-4y^2}\right)\right]/\left[\frac{1}{2}-y-\left(\frac{1}{2}+y\right)\exp\left(\frac{8yM}{1-4y^2}\right)\right], \text{ when } x<1/2$$

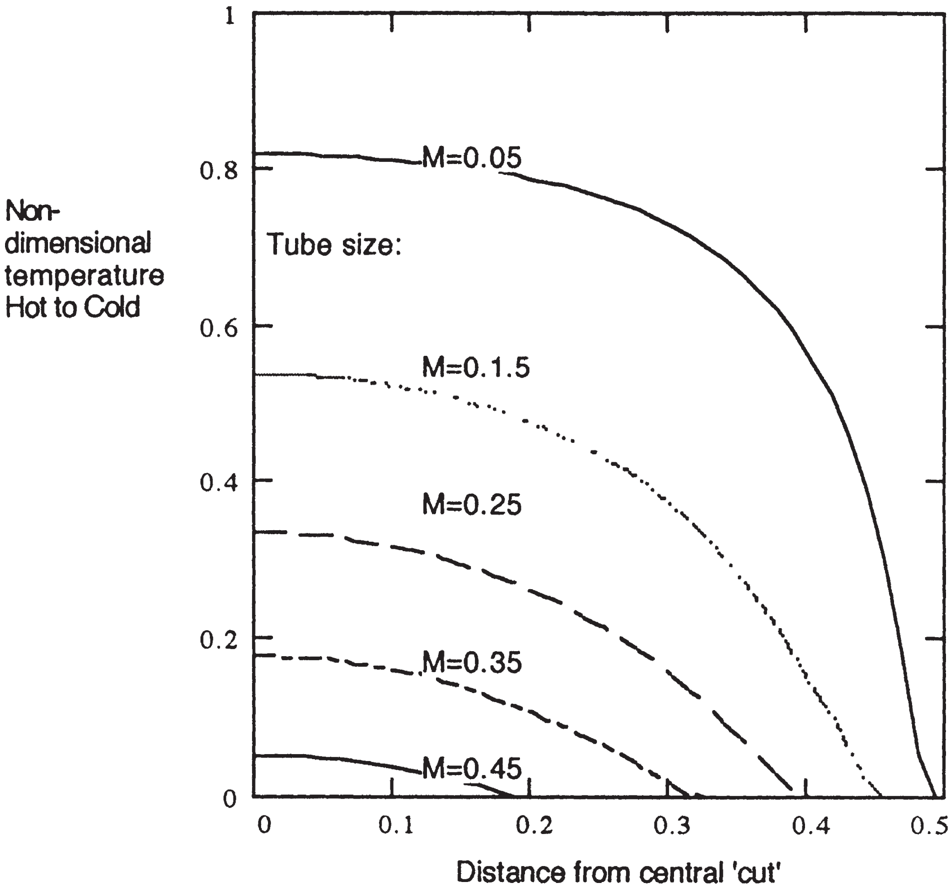

Fig. 4. The tube temperature difference as a function of cold cut.

[2] Outside the working length, the tube performance can also degrade by heat transfer to or from the walls.

[3] The function returns to zero at the end of the curve, which is physically sensible. The portion below zero might have a role to play in representing the anomalous behavior reported in the literature [3] where the inner core exits *hotter* and the annulus *cooler*.

$$\frac{\varepsilon}{x}=\left[1-\exp\left(\frac{8yM}{1-4y^2}\right)\right]\Big/\left[\frac{1}{2}+y-\left(\frac{1}{2}-y\right)\exp\left(\frac{8yM}{1-4y^2}\right)\right],\ \text{when } x>1/2, \tag{12}$$

which is seen to be symmetric about the origin in y. An expansion in powers of y shows the continuity, flat center and central maximum:

$$\Delta T_{\max}=\frac{4M}{1+2M}\left[1+16\left(\frac{2}{3}M^2+\frac{1+2M^3}{1+2M}\right)y^2\right] \tag{13}$$

If we assume a satisfactory design, the size M of the vortex tube as a heat exchanger will be small and we may expand the expressions[4] for the effectiveness to obtain

$$\varepsilon=\frac{M}{x}\left[1-\frac{M}{2x(1-x)}\right],\ x<\frac{1}{2}\ \text{ and }\ \varepsilon\cong\frac{M}{1-x}\left[1-\frac{M}{2x(1-x)}\right],\ x>\frac{1}{2} \tag{14}$$

However, it turns out that equal cut is wanted for the second optimization for which the explicit solution should be used.

We also take the simple view that the effective length L is independent of the cut. This is unlikely to be exactly true; however, the determination of the variation of the effective length is a matter for detailed computational fluid dynamics.

4. Second optimization

We suppose that the first optimization is carried out in the temperature ratio $\Delta T/T_0$ so that

$$q_{\text{opt1}}/T_0=x(1-x)(1-\Delta T/T_0)^{1/2}[1-(1-\Delta T/T_0)^{1/2}]=(1/2)x(1-x)\Delta T/T_0= \tag{15}$$
$$(1/2)x(1-x)(\Delta\theta/T_0)[1-\varepsilon/(1-x)],\ \ x<1/2,\ \text{first order.}$$

At first sight, this looks like a complicated optimization over the cut because both $\Delta T/T_0$ and $x(1-x)$ depend upon the cut x. Note that both, separately, are maximized by taking a cut of one half. We also showed maximum in $\Delta T/T_0$ up to 3/4 was advantageous. Hence we employ the explicit effectiveness expression and obtain

$$\frac{q_{\text{opt1}}}{T_0}=\frac{1}{4}\left(1-\frac{\Delta\theta}{T_0}\ \frac{1-2M}{1+2M}\right)^{1/2}\left[1-\left(1-\frac{\Delta\theta}{T_0}\ \frac{1-2M}{1+2M}\right)^{1/2}\right]\cong\frac{1}{8}\ \frac{\Delta\theta}{T_0}\ \frac{1-2M}{1+2M} \tag{16}$$
$$\left(1-\frac{1}{4}\ \frac{\Delta\theta}{T_0}\ \frac{1-2M}{1+2M}\right),\ \text{two orders.}$$

[4] For given x these expressions are valid for small enough M but we may not then allow y to go to unity or zero in the result of fixed M.

Fig. 5 shows the variation of first optimized refrigeration with heat exchanger size and cut. It shows that for high efficiency a low heat exchanger size is desirable, and that the second optimization over the temperature difference is flat and insensitive to the cut. However in all cases the leading term in $x(1-x)$ is quite sharply peaked and therefore overall the optimization is sensitive to the cut. Fig. 5 gives the performance in terms of the nondimensional refrigeration $q_{1,\text{optimum}}/T_0$ over cut x and heat exchanger size M, showing the 'peakiness' of the optimization with cut.

We have therefore found in this model a way to second-optimize the refrigeration effect of the Ranque–Hilsch tube and regenerative heat exchanger system.

5. Second law bounds

The Second Law of Thermodynamics enables us to put some limits on the various quantities we have formulated. The intrinsic temperature difference of the tube $\Delta\theta$ goes as $\Delta\theta\leq$

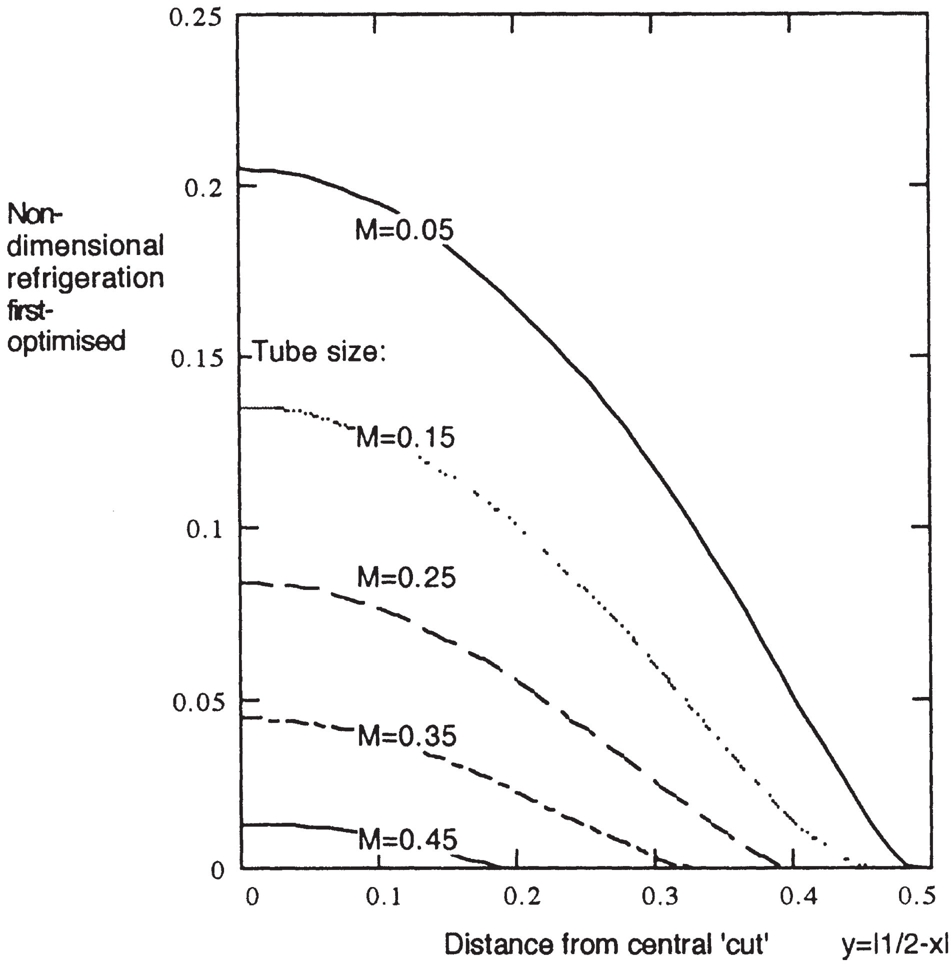

Fig. 5. First-optimized nondimensional refrigeration versus cut and size.

$(p_1/p_0)^{\gamma-1/\gamma}-1\equiv\Gamma-1$ but this is not a sharp bound as $\Delta\theta$ is only an order of magnitude expression. Nevertheless, the relationship suggests how the temperature difference and hence refrigeration powers would go with different gases. For example, at a pressure ratio of 5, helium is a factor 6.5/5.6 better than air. A sharper bound based on measurable temperature differences is for ΔT. For the optimized cut we have an entropy balance for the vortex tube alone as

$$\dot{S}_{\rm gen}/(\dot{m}c_{\rm p})=\ln\Gamma(1-x)\ln(T_{\rm H}/T_0)+x\ln(T_{\rm C}/T_0)=\ln\Gamma+(1-x)\ln(1+x\,\Delta T/T_0)+ x\ln[1-(1-x)\Delta T/T_0]\geq 0. \tag{17}$$

Expanding to second order (there is no entropy generated for vanishingly small temperature differences) gives a bound

$$\Delta T\leq T_0\left[\frac{2\ln\Gamma}{x(1-x)}\right]^{1/2} \tag{18}$$

or for the optimum cut,

$$\Delta T\leq T_0(8\ln\Gamma)^{1/2}\text{ and, correspondingly, } q_{\rm opt12}\leq T_0[(\ln\Gamma)/8]^{1/2}. \tag{19}$$

We note that the combination of heat exchanger and vortex tube provides no sharper bound because the entropy increase in the regenerative heat exchanger is positive and already offsets the entropy reduction due solely to temperatures in the tube; the entropy increase due to the pressure drop is not essential in the combination. For example, with no heat pumping, the system returns the flows at ambient temperature, with no net entropy change save that due to pressure drop.

6. Conclusions

Simple models, that include perfect gases, order-of-magnitude analysis for the vortex tube as a heat exchanger and the use of regenerative and refrigerative heat exchangers of unit effectiveness, allow an optimization of the vortex system as refrigerator and an estimate of the optimum refrigerative effect if the leading parameter of the tube, the temperature difference established at equal cut can be measured or calculated. It is notable, to first order in the temperature $\Delta T/T_0$, that the first optimum calls for a load half way from zero to the maximum refrigerative effect available at a given cut, and the second optimization has this cut best at one-half, i.e. balanced hot and cold streams. To second order in temperature, the first optimization is just less than half the maximum cooling effect. We were also able to provide bounds for the temperature difference that could be established from a given pressure ratio and gas.

The optimizations do not depend upon any explicit model and thus do not predict the actual performance but rather where it will be optimized, in cut and load. More exact results would seem to turn on more detailed theory which might well necessitate computational fluid dynamics. One extension we have considered (and can develop readily within the framework of the simple model) is the idea of cascades of such vortex systems, ultimately leading perhaps to liquefaction of all or part of the gas employed. However, such an extension would strain the applicability of the perfect gas assumption, to be treated another time.

Acknowledgements

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